Lecture 5: **Conservation Laws**

- Newton's laws give rise to two important conservation laws in Classical Physics: **conservation of momentum and energy**.
- Momentum can be linear or angular. In this course we will only consider linear momentum.

Momentum, energy and angular momentum cannot be created or destroyed.

Conservation of momentum

Suppose no forces act on a system (F=0). Then

$$\frac{d}{dt}(mv) = 0$$

$$mv = \text{constant}$$

This is known as the *law of conservation of momentum.*

- In a closed system (i.e. no external forces) momentum is always conserved.
- **Force** is the **time derivative of momentum.** Therefore the concept of force can be replaced by that of momentum.

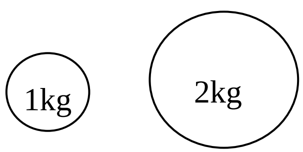
Using conservation of momentum to solve collision calculations

The differential form of Newton's Laws holds at every point along a trajectory: to determine an outcome it is necessary to know the force at every point.

However, the principle of conservation of momentum lets us determine the condition "after" in terms of the conditions "before" - without knowing in detail what has happened in between (e.g. collisions)

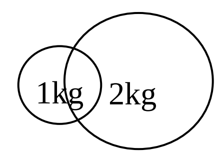
Example:

BEFORE



→ $10\ ms^{-1}$ stationary

AFTER



→ $3.33\ ms^{-1}$

- Momentum before = 10 kg ms-1

Momentum after = 10 kg ms-1

Conservation of Energy

The principle of conservation of momentum is very useful but it does not let us look inside a system where the force is non-zero.

There is another conservation law that holds when forces are acting: conservation of energy.

Consider a particle that moves a distance *s* under a constant force *F*.

Then, from kinematics and *F=ma*, we can write:

$$v^2 = u^2 + 2\frac{F}{m}s$$

$$Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

Define: $W = Fs$ the ***kinetic energy***

$T = \tfrac{1}{2}mv^2$ the ***work* done**

then $W = \Delta T$

This is the Work-Energy theorem: The change in a particle's kinetic energy between two points is equal to the work done on the particle between those two points.)

Physical significance of *W*

The *work* done by a force is a measure of the energy used or provided by that force.

In general terms,

$$W = \int F dx$$

$$= Fs \qquad \text{for constant } F$$

When a force does not move, *no work is done*.

Work is measured in **Nm** or **Joules (J)**

The *rate* of doing work is called the *Power*.

The unit of power is the **Watt** **(1W = 1 J s^{-1})**

Physical significance of *T*

The Kinetic Energy of a moving object is equal to the work done in accelerating it from rest.

The Work-Energy theorem is reversible: just as work can be used to accelerate an object (increase its KE), so a moving object can do work in slowing down to rest.

Thus, the KE of an object can be considered as its *capacity to do work* by virtue of its motion.

This gives rise to the concept of energy as “stored work”.

This concept is particularly useful when other forms of energy are defined.

Energy is *not* the same as work even though we use the same unit for both (the Joule).

Power (the rate of doing work) can be defined as the rate of change of the energy of an object.

Obtaining a Conservation Law

The work-energy theorem : $W = \Delta T$

is not a conservation law and hence of limited usefulness.

To make it into a conservation law, we need to write *W* as the difference of two terms V_1 *and* V_2, so that:

$$-\Delta V = \Delta T$$

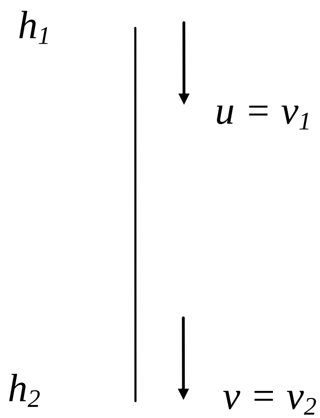
(note the – sign: this is so that we can write $\Delta V + \Delta T=0$)

V is called *potential energy* as it has the capacity to be changed into kinetic energy, and hence into work.

Clearly, *V* is some function of the force.

A general derivation will not be given here, but we will take a special case: a particle moving under a constant force (eg. gravity close to the surface of the Earth).

If a particle falls from a height h_1 to a height h_2



$$W = mg\left(h_1 - h_2\right)$$

$$mgh_1 - mgh_2 = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

If we associate a function:

$$V(h) = mgh$$

called the gravitational *potential energy*, with the force of gravity, then:

$$V(h_1) - V(h_2) = T(h_2) - T(h_1)$$

which is what we need.

We can write the above result as:

$$-\Delta V = \Delta T$$

$$\Delta(T+V) = 0$$

$$E = T + V = \text{constant}$$

E is called the *total mechanical energy* and is conserved in a closed system

Conservation of energy implies that although the balance between *T* and *V* can change, **their sum is constant**. Thus *V* can be considered a form of energy due to position, which is converted to kinetic energy (motion) and back again.

The potential energy function (called a **field**) summarises all that is known about the force. If the potential energy function is known, then the motion at an end point can be calculated in terms of the motion at the starting point - without knowing the details of the motion in between.

Describing physical systems in terms of potential energy is therefore a very useful tool when the description needs to be as general as possible.

This is a general result. Here's how to understand it.

The key step is associating a potential energy function with the force. We need:

$$V(b)-V(a)=-\int_{x=a}^{x=b} F\,dx$$

If this holds, then clearly the work done in going from any point back to the same point, *via any path*, is zero.

We write: $$\oint F\,dx = 0$$

integral along a closed path

and the force is called a ***conservative force*.**
Clearly, gravity is a conservative force.

Mathematically, what is required to be able to define a potential is that the force be a *single-valued function of position*.

- Is friction a conservative force?

- Do there exist non-conservative forces?

- Is energy always conserved?

Potential energy

From the definition of potential energy

$$V(b) - V(a) = -\int_{x=a}^{x=b} F dx$$

we can obtain:

$$F = -\frac{dV}{dx}$$

Therefore, instead of considering forces (Newton) we can consider potentials (the **Action Principle** of Lagrange, Hamilton, ….)

Potential is a *scalar* while force is a *vector.* This can be useful since it a simpler mathematical quantity.

Strictly speaking, we can never talk about potential - only *potential difference*.

However, in practice, if the potential at point *a* is defined as 0, we can say that *V(b)* is “the potential” at point *b*.

Conservation laws

The principles of conservation of momentum and energy *go beyond* classical mechanics - they also hold in relativity and also in quantum mechanics, where the state "in between" is not only unknown but unknowable.

It is quite possible to do the derivations in reverse. Starting from the principle of conservation of momentum, it is possible to *derive* Newton's IIIrd law. The IInd law can be derived as a consequence of the conservation of energy.

- In fact, **conservation laws** are held to be **the** fundamental laws of nature. They are related to **symmetries** that describe the behaviour of the Universe **(Noether's Theorem)**

The Universe has **translational symmetry**: the laws of physics are the same at any point in space. This implies conservation of momentum.

The Universe also has **symmetry in time** : the laws of physics are the same at any point in time. This implies conservation of energy.

Other conservation laws (angular momentum, charge, parity …) reflect other symmetries.

Symmetry groups

Group	Transformation	Unmeasurable Quantities	Conserved Quantities
Rotation, SO(3)	Spatial rotations	Absolute angle	Angular momentum L
Translation	Spacetime translations	Absolute position	Energy E, or Mass M and Momentum P
Lorentz	Spacetime rotations and Reflections	Absolute uniform velocity, Orientation	Spacetime interval S, Parity P, Time reversal T
SL(2,C) (Homogeneous Lorentz)	Spacetime rotations	Absolute uniform velocity	S (not P or T)
Diffeomorphism (General Coordinate)	Spacetime curvature (acceleration)	Absolute acceleration	Topological invariants*
Poincaré	Lorentz plus Translations	(see above)	L, E (or M) and P
U(1)	Scalar Phase Shift	Absolute phase	Electric charge
SU(2)	2-D Phase Shift	Absolute 2-D phase	Isospin
SU(3)	3-D Phase Shift	Absolute 3-D phase	Color

Relax. This is way beyond the stipulated course content. Definitely not exam material!