Towards a Gauge Theory of Gravity

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Fundamental Interactions

- All physical interactions in nature described by 4 fundamental forces:
  - Gravity
  - Electromagnetism
  - Weak nuclear force
  - Strong nuclear force

- EM, WNF, SNF are described by quantum theories

- Forces are subject to Heisenberg's Uncertainty Principle

- Not all field strengths at any given point can be specified with arbitrary precision

However…

- No quantum theory of gravity exists

The Gauge Principle

- Gauge theories for:
  - Electromagnetism
  - Weak nuclear force
  - Strong nuclear force

- Gravity can be formulated as a gauge theory

- Gauge Principle:

  Physical system invariant with respect to global group of continuous transformations remains invariant when group considered locally. Partial derivatives are replaced by covariant ones, which depend on some new vector field.

- These fields correspond to four known forces

Electromagnetism as gauge theory

- Electromagnetic interactions arise from demanding invariance of quantum wave equation under local phase changes
- Perform phase transformation $\psi \rightarrow \psi' = \psi e^{i\alpha}$
- We can modify Schrödinger equation to make it invariant under local phase changes
- Changes in a particle's phase result in changes to its momentum
- There must be a force which performs these changes
- That force is electromagnetism

General Relativity

Physical Theory with the following properties:

- Spacetime represented by 4-dimensional differentiable manifold
- Manifold is equipped with Lorentz metric giving spacetime distance between points
- Physical models consist of manifold, metric, and matter fields, which satisfy Einstein's Equation

$$G^{ab} = \kappa T^{ab}$$

- Spacetime itself defines the field

Gauge Theory Gravity (GTG)

- Motivated by attempt to find Theory of Gravity which follows gauge principle
- Developed by Anthony Lasenby, Chris Doran, Stephen Gull (Cavendish Laboratory, Cambridge), 1990s
- Aim: To model gravitational interactions in terms of gauge fields in the spacetime algebra
- Choice of geometric algebra as mathematical language
  - Clifford's generalisation of Hamilton's quaternion algebra (1878)
  - Discovered independently by Grassmann (1877)
  - Developed by David Hestenes (1966) – *Spacetime Algebra (STA)*

Geometric Algebra

Clifford Algebra:

- Utilise vectors with single associative product distributive over addition
- Geometric product ab
- Inner product $a \cdot b \equiv \frac{1}{2}(ab + ba)$
- Outer product (bivector) $a \quad b - \frac{1}{2}(ab \quad ba)$
- Geometric product is sum of scalar and bivector part $ab = a \cdot b + a \wedge b$

Spacetime Algebra in GTG

- Utilise set of 4 orthonormal basis vectors satisfying $\gamma_\mu \cdot \gamma_\nu = \eta_{\mu\nu}$
- Set of 4 independent basis vectors for spacetime, giving 16-dimensional spacetime algebra
- Each point in vector space represented by vector $x = x^\mu \gamma_\mu$
- Background spacetime is global, as in other gauge theories

Gauge Principles for Gravitation

- All physical fields have generic form $A(x) = B(x)$

A and B are spacetime fields
x is a spacetime algebra position vector

1. **Position Gauge Invariance**

- Statement is independent of where we place the fields in spacetime algebra, i.e..
If $x = f(x)$ then $A(x) = B(x)$ has same physical content.

2. **Rotation Gauge Invariance**

- Intrinsic content of relation $A(x) = B(x)$ at a given x_o must be unchanged if we rotate A and B by same amount, i.e..
$A(x) = RA(x)$ and $B(x) = RB(x)$ does not alter physical consequences.

Physical interpretation:

- Absolute position and orientation of particles and fields in the spacetime algebra is not measurable
- We only extract relative relations between fields
- These relations must be local
- This ensures no conflict with the principles of General Relativity
- Theory is locally equivalent to GR

Field Equations in GTG

- Ricci tensor: $R(b) = \partial_a \cdot R(a \wedge b)$
- Ricci scalar: $R = \partial_a \cdot R(a)$
- Einstein tensor: $G(a) = R(a) - \frac{1}{2} aR$
- Einstein Equation: $G(a) = \kappa T(a)$

Gravitation in GTG

- We need a field which guarantees gauge invariance
- Introduce new tensor field h^{-1} and its adjoint h
- Maps h^{-1} change the "metric" of spacetime
- Space ***H*** formed by maps plays role of curved spacetime
- "Gravitational force" is expressed via this field acting on flat spacetime

GR and GTG compared

- GTG is locally equivalent to GR if spin effects are ignored
- GR is recovered in the zero-spin limit
- Spin density is usually irrelevant on gravitational scales
- Theories agree in the Newtonian limit and Classical Tests of GR (Light deviation, perihelion shift, time delay, gravitational redshift)
- Theories shown to agree for most important physical models: static & rotating spherically symmetric, cylindrically symmetric matter distributions

Advantages of GTG

- It is possible to treat gravitation within the framework of the other theories of fundamental interactions

- GTG does not allow "non-physical" solutions of Einstein's Equations (e.g. wormholes) due to global nature of gauge theory
- No empirical evidence for such solutions

- Difficulties with $k \neq 0$ cosmological models because of global nature of curvature
- Current evidence indicates $k = 0$

Experimental distinction between theories

- Differences emerge over issues such as:

- Global topology
  - Treatment of black hole singularities
- Interaction with quantum theory due to spin effects

BUT

- Classical tests of GR carried out in free space, where no spin is present

- No way to observe black hole thermodynamics yet

Current Work

- **Finding an experiment which will differentiate between GR and GTG**

Possible candidates:

- Time asymmetric processes
- Black hole thermodynamics (Hawking radiation)

- Quantum spin effects
- Quantum effect over regions of spacetime separated by event horizon

- Use of linearized theory in GTG
- Fields acting on flat spacetime, unlike GR

Difficulties with linearized theory

- Add small perturbation to flat spacetime: $g_{ab} = \eta_{ab} + p_{ab}$

- Obtain vacuum solution: $\left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3} - \frac{1}{c}\frac{\partial}{\partial t}\right)\bar{p}_{ab} = 0$

- Gravitational waves predicted by GR

- How do perturbations propagate in GTG?
- Is adding term to metric equivalent to adding gravitational potential?

Linearized theory in GTG

- Method must be modified since $g_a = h^{-1}(e_a)$

- Perturbation has the form $g_a = h^{-1}(e_a) + p^{-1}(e_a)$

  where both functions follow gauge principle

- Ignore second-order terms when calculating $g = g_a \cdot g_b$

- Wave solution has same form as GR

Further Work

- Exact solutions for gravitational waves
- Gravitational waves in "curved background spacetime"
- Solutions in strong-field region
- Comparison with GR results

Towards a gauge theory of gravity

Four fundamental interactions, following gauge principle

One single theory?