INTERFERENCE - II

When two (or more) *coherent* waves pass through the same volume of space, the principle of superposition predicts a stable interference pattern. In general, the waves will be identical except that they will have travelled a different distance to reach a given point (there is a *path difference D*)

$$y = A\left[\cos(\omega t - kx)\right.$$

Using the trigonometric identity:

$$\cos a + \cos b = 2\cos$$

the above equation can be written:

$$y = 2A\cos\left(\frac{1}{2}k\Delta\right)\cos$$

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This represents a wave:

- that has a maximum amplitude of 2A

- that has the same frequency and wavelength as the two original waves, and a phase equal to the average of the two;

- that the amplitude is *modulated* by a spatial function of the path difference.

This modulation term can be written:

$$M = \cos\frac{k\Delta}{2} = \cos\pi\frac{\Delta}{\lambda}$$

Thus, the amplitude is:

- a maximum when the path difference is zero or an integral number of wavelengths (constructive interference),

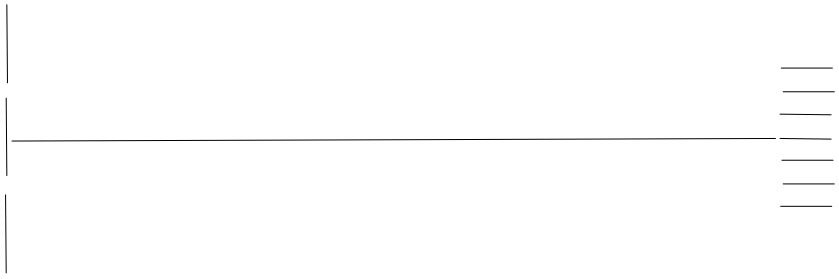
- zero when the path difference is an odd number of half-wavelengths (destructive interference).

The double-slit experiment

This classic interference experiment was first performed on light by Thomas Young, thus proving conclusively that light is a wave motion.

two closely-spaced slits illuminated by the same light source

pattern of equally spaced bright and dark "fringes"

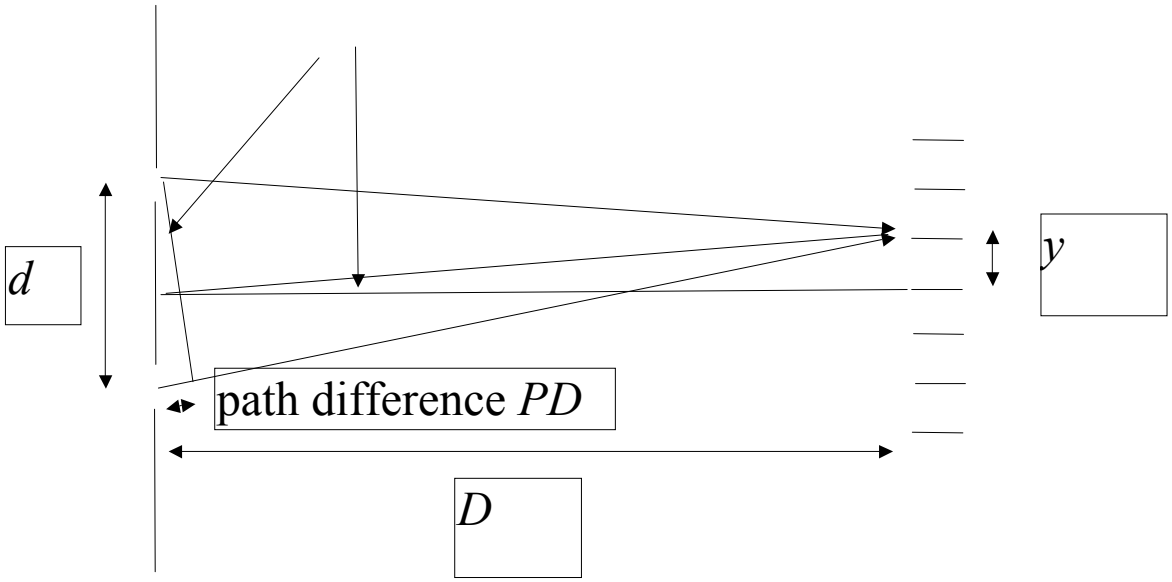


NOTE: black line represents a bright fringe

The reason for the bright central fringe is easy to understand since, being opposite the midpoint of the two slits, waves travel an equal distance to reach it and so interfere constructively.

The "path difference" formula obtained above can be used to derive the spacing between the bright fringes.

same angle q



$$PD = d\sin\theta \approx d\frac{y}{D}$$ for small q

so:

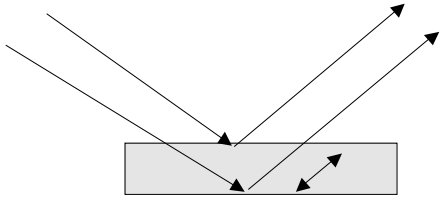
$$n\lambda = d\frac{y}{D}$$

$$\boxed{y = n\lambda\frac{D}{d}}$$

Note that the *smaller* the slit spacing, the *wider* the fringe pattern. Note also the dependence on frequency, which requires that the experiment be done with monochromatic (single-colour) light.

Practical examples of interference:

- colours in oil films and soap bubbles. Constructive interference occurs at different angles for light of different wavelengths.



path difference (note must take refractive index of film into account).

- fading in radio/TV reception when an aircraft flies overhead (multipath distortion)