ELECTROMAGNETISM

- James Clerk Maxwell (1831-1879)

- With Newton's Gravitation and the Laws of Motion, provided a *complete* description of the physical world as understood at that time (classical physics)

- Unifies electrical and magnetic phenomena: driven by the discovery of the laws of induction by Michael Faraday (1791-1867)

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- Is the first *field theory* in physics: provides explanation for action-at-a-distance in terms of fields.

- Predicts existence of electromagnetic waves

- Provides the basis for Einstein's Special Theory of Relativity

1. Electric Charge

ELECTRIC CHARGE is a property of matter (like mass)

It is measured in *Coulombs* which is defined in terms of forces between conductors carrying moving charges.

It comes in two kinds (called positive and negative). All objects contain charges but normally have zero net charge. However, it is possible to separate these charges, and perform measurements on a body with a net charge.

Positive and negative charges attract each other.

The law governing the electric force between two charges is similar to that governing the electric forces between two masses:

$$F_{\text{repulsion}} = \left(\frac{1}{4\pi \varepsilon_0} \right) \frac{q_1 q_2}{r^2}$$

Note however that since either q can be negative, the force can be either attractive or repulsive.

The constant of proportionality has the value $9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$ (compare $G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$)

2. The Electric Field

In the same way as we defined the *gravitational field strength* **g** as the force on unit mass, it is convenient to define the *electric field strength* **E** as the force on unit charge. Thus the value of the electric field at a distance r from a charge q_1 is given by:

$$\mathbf{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1}{r^2}$$

where **E** is a vector that points *away* from a positive charge or *towards* a negative charge.

The force on a charge q_2 at a point where the electric field is **E** is thus:

$$\mathbf{F} = \mathbf{E}q_2$$

which also has the same direction as **E**, unless of course q_2 is negative, in which case it has the opposite direction.

The force on a charge due to a collection of charges is the sum of the forces due to each individual charge in the collection. Similarly, the electric field due to a collection of charges is the sum of the individual electric field contributions of each charge.

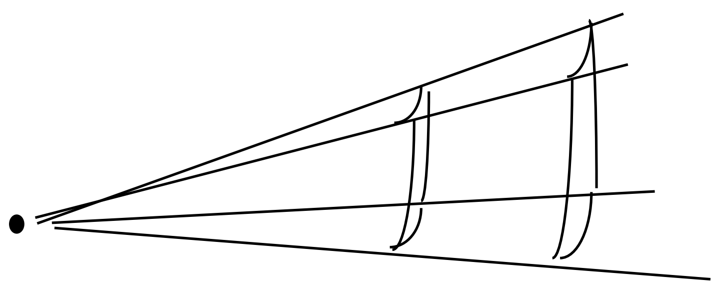
Since force (or electric field) are vector quantities, this summation may not be simple.

3. Gauss's Law

The inverse-square law followed by the electric force (and gravitation) has an interesting consequence.

The *flux* ϕ of a field is defined as the product of the perpendicular component of the field passing through a surface, and the area of the surface.

Since the surface area of a sphere (or section thereof) is proportional to r^2 while the filed goes as r^{-2}, the flux through a surfaces bounded by the same radii is constant.



In particular, the flux through a complete sphere:

$$\phi_E = \left(4\pi r^2\right)\left(\frac{q}{4\pi\varepsilon_0 r^2}\right) = \frac{q}{\varepsilon_0}$$

It can be shown that this holds for any closed surface **S**, giving:

$$\boxed{\oint_S \mathbf{E}.d\mathbf{S} = \frac{q}{\varepsilon_0}} \qquad \text{.... I}$$

4. The Electric Potential

Since like charges attract, work (F x d) is done in separating them. This work then goes into the *electric potential energy* of the separated charges.

If a charge moves along the direction of a constant electric field, the work done *by the electric field* is:

$$W = Fd = Eqd$$

If a charge q moves from position d_1 to position d_2, then the work done is:

$$\Delta W = -q\left(Ed_2 - Ed_1\right) = -Eq\left(d_2 - d_1\right)$$

where the negative sign reflects the fact that a positive charge will *gain* energy (have work done upon it) when it moves in the same direction as E.

We can thus define the *electric potential* V at point in this field by:

$$V\left(d\right) = -Ed$$

where a charge q will gain energy $q\Delta V$ when it moves between points at a potential difference ΔV.

Electric potential is measured in a special unit called the *volt:*

$$1\text{V} = 1\text{ J C}^{-1}$$

It is usually more convenient to measure potential ("voltage") than electric field, the latter is more usually expressed in Vm^{-1} than in the equivalent Nm^2C^{-1}.

The above argument was performed for a uniform field. The field around a point charge is of course not uniform:

$$\mathbf{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1}{r^2}$$

so an integral has to be taken:

$$\begin{aligned} V &= -\int E\,dr \\ &= -\int \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1}{r^2} \\ &= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1}{r} \end{aligned}$$

The necessary constant of integration is ignored by the convention that the potential at large distances ($r\rightarrow\infty$) is zero. A positive charge near another positive charge will have positive potential energy and thus tend to move away.

Like the gravitational field, the electric field is *conservative*, meaning that the work done in moving between two points is equal and opposite to doing the trip in the reverse direction. This is necessarily so because:

(a) conservation of energy requires it

(b) from the above definition of V, moving a charge from any point back to the same point has a net potential energy difference of zero

This can be written in mathematical notation:

$$\oint \mathbf{E}.d\mathbf{l} = 0 \qquad \text{.... } \mathbf{IIa}$$

where the symbol means taking the integral of the electric field over any closed loop in space. This formula is particularly important in understanding the movement of electric currents, since it implies that the total potential round a loop (and a circuit is a loop) is zero.

As will be noted later, this is not strictly correct since it does not take into effect of magnetic fields.

5. Electric Current

When electric charges move (from high potential to low potential), the result is called an *electric current.*

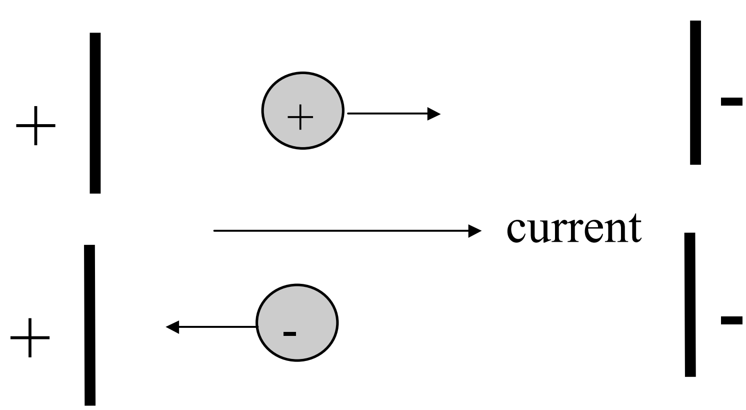
$$I = \frac{dq}{dt}$$

The unit of electric current is the *Ampere*:

$$1\text{A} = 1\text{C s}^{-1}$$

(Actually, the definition is the other way round: the fundamental unit is the Ampere).

The charges move in response to an electric field, ie. between places of different potential. Note that current always flows *down* a potential difference, from high potential to low potential. This is irrespective of the sign of the charges that make up the current.



The relationship between the current I that flows between two points at potential difference ΔV depends on the *resistance* to the flow of charge offered by the medium between the two points. This relationship can be very complicated because of microscopic processes within the medium, which in turn depends on the nature of the medium.

For metals at relatively low currents, a simple proportion does exist:

$$I = \frac{1}{R} V$$

This is known as **Ohm's Law**.

When a charge flows down a potential gradient, it does work (loses energy). This energy is normally evidenced as heat within resistances. The rate at which this energy is produced is given by:

$$P = \frac{dW}{dt} = \frac{d}{dt} Vq = IV$$

This energy, of course, comes from the source of the electric field which moved the charge to the position of high potential in the first place.