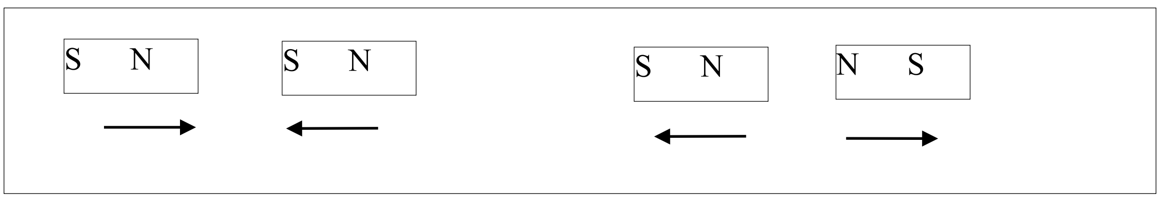
ELECTROMAGNETISM (cont'd)

6. Magnetic effect of current

By "magnetism" we mean the force by which "magnets" attract or repel each other, and by which a compass needle points to the Earth's pole.



The situation is superficially similar to the attraction and repulsion between charges so we would like to write a similar set of equations:

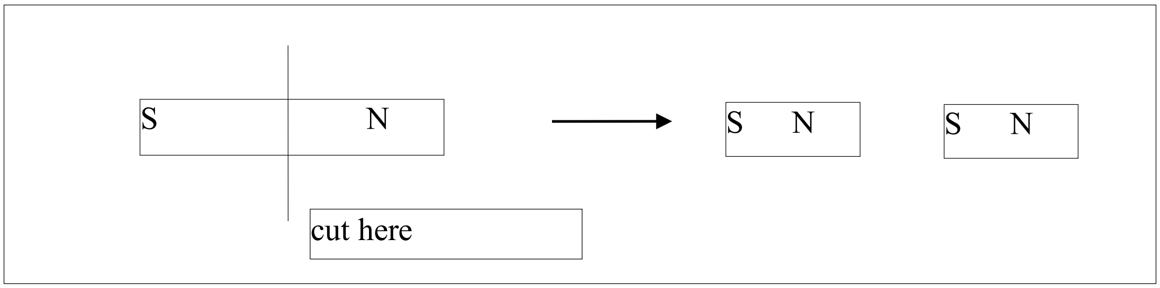
$$F^M = \frac{1}{4\pi\mu_0}\frac{q_1^M q_2^M}{r^2}$$

$$F^M = B\, q_2^M \qquad B = \frac{1}{4\pi\mu_0}\frac{q_1^M}{r^2}$$

$$\oint_S B.dS = \frac{q^M}{\mu_0}$$

where q^M is the magnetic equivalent of charge.

Unfortunately, this cannot be done because magnetic charges ("monopoles") do not exist – or, at least, have not yet been discovered.



Magnetic poles always come in pairs – magnets are *dipoles*.

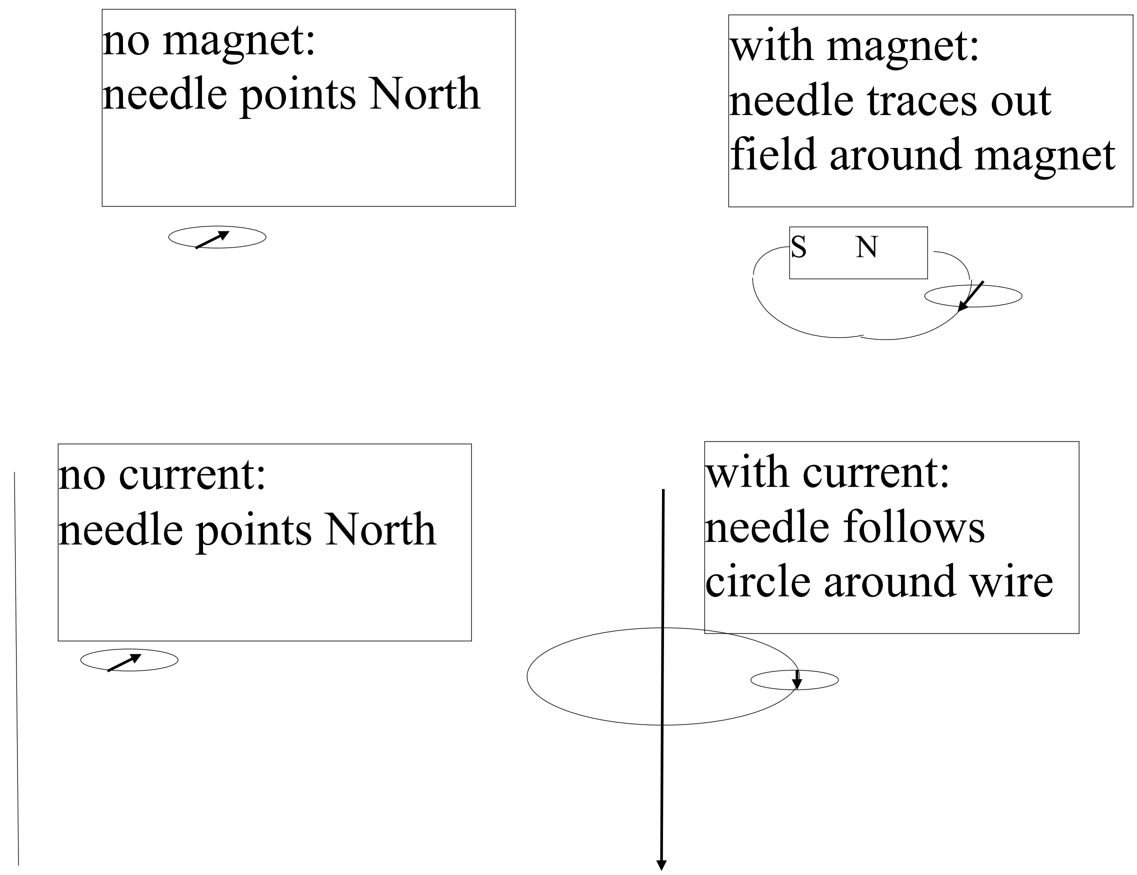
We can express this fact mathematically as:

$$\oint_S \mathbf{B}.d\mathbf{S} = 0$$

Electric dipoles also exist and the mathematics of dipole fields is well established. However since isolated magnetic poles do not exist, it does not make sense to develop a theory based upon them.

We therefore need to find another starting point for developing a theory of magnetism.
The link between magnetism and electricity is obtained by noting that an electric current deflects a compass needle in

a similar way to a magnet, showing that an electric current generates a magnetic field.



The *strength* of the magnetic field due to a wire carrying a current *I* is found to follow a simple law if the wire is very long compared to the distance *r*:

$$B = \frac{\mu_0 I}{2\pi r}$$

where *I* is the current and *r* the distance from the wire. The constant m_0 is given the exact value 4p x 10^{-7}. This can be done because *B* is being defined in terms of *I*, which is also defined in terms of electromagnetic forces (later).

Note also that this definition of *B* is identical to that which was given in the first slide (stated without proof).

This equation defines the unit of magnetic field (the *Tesla, T*) in terms of the field produced a distance of 1m away from a long wire carrying a current of 1A.

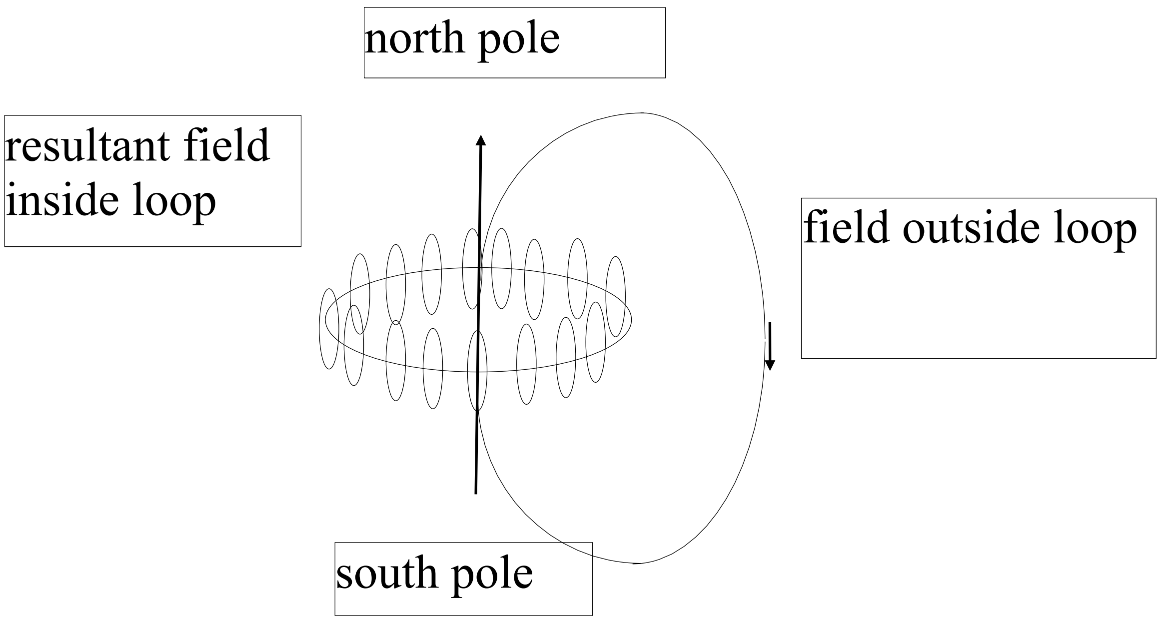
Notice that the denominator of the above equation is simply the circumference of a circle. This gives rise to another mathematical expression for the magnetic field, known as Ampere's Law:

$$\oint \mathbf{B}.dl = \mu_0 \mathbf{I} \qquad \textbf{... IIIa}$$

Today, we understand that electric currents are the only source of magnetic fields and that "magnets", and indeed the Earth itself, have electric currents circulating within them that give rise to their magnetic fields.

7. Magnetic poles

Electric currents normally only flow in complete circuits. Hence, even a long straight wire is really only part of a *current loop* .

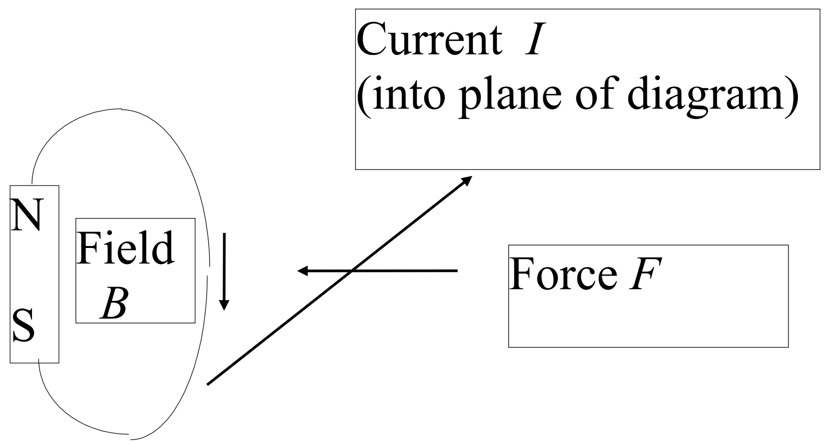


Since a loop is a 2-D entity that has 2 faces, magnetic "poles" always come in pairs. There are therefore no magnetic equivalents to charges. As seen previously, Gauss's Law thus for magnetic field thus becomes:

$$\oint_S \mathbf{B}.d\mathbf{S} = 0 \qquad \textbf{.... IV}$$

8. Magnetic Force on a current

When a current-carrying wire is placed in a magnetic field (however generated), it experiences a force. This is known as the *motor effect*.



The magnitude of this force (per unit length of wire) is simply the product of the magnetic field and the current:

$$\frac{F}{l} = B\,I \qquad\qquad F = B\,I\,l$$

(There is no constant of proportionality because of the way the unit of *B* us defined)

Note the peculiar arrangement of the direction of the force: it is *perpendicular* to both the field and the current, which must in turn be perpendicular (or have components perpendicular to) one another.

Note that when a single charge q moves with velocity v in a magnetic field, it also experiences a force. This is how the force on the wire arises, as the sum of the force on all the moving charges inside it.

We can obtain the equation for this force by noting that:

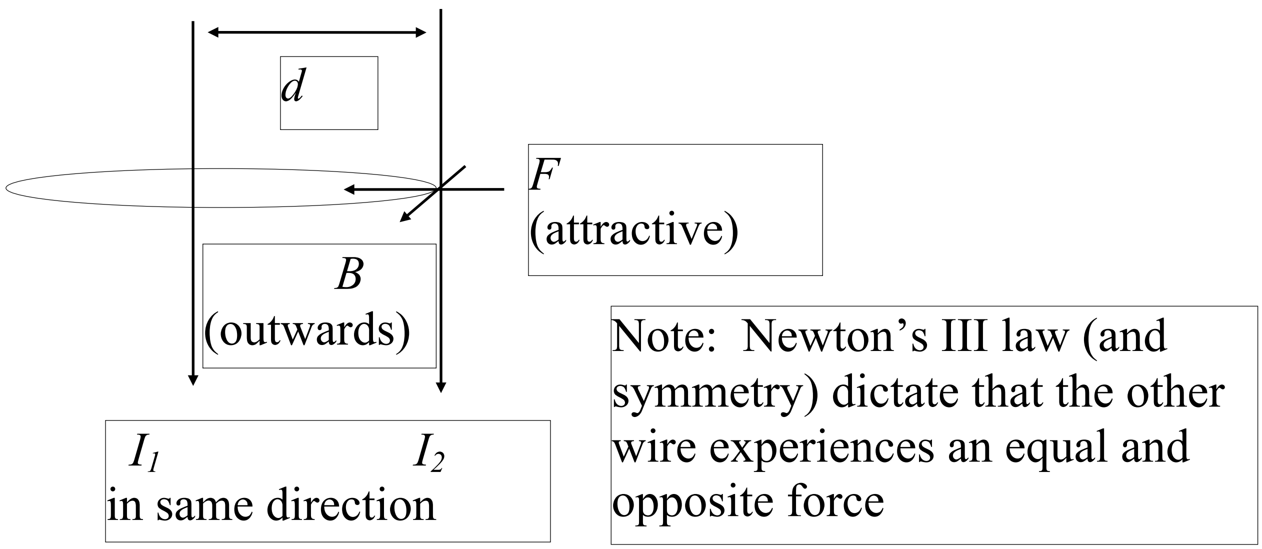
$$I = \frac{dq}{dt} \qquad\qquad v = \frac{dl}{dt}$$

$$F = BIl = B\frac{d}{dt}ql$$

$$\boxed{F = Bqv}$$

From a philosophical viewpoint, this is the fundamental equation and the equation for the force on a current-carrying wire should be derived from it. Historically, however, it was possible to measure forces on wires long before individual charged particles were identified.

When the magnetic field is caused by another long wire, then the force between the wires depends only on the currents and the distance between them, without involving the “magnetic field” at all.



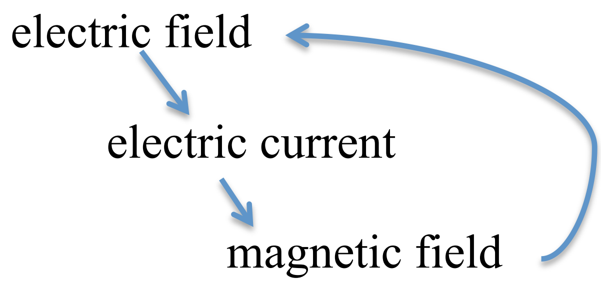
$$\frac{F}{l} = BI_2 = \frac{\mu_0 I_1 I_2}{2\pi d}$$

This is in fact how all the units described above are defined in terms of the usual measureables of length, mass, force, etc. The resulting equation defines the ampere in terms of the newton and the metre and hence all the other units.

9. Electromagnetic induction

The phenomenon that "closes the loop" beween electricity and magnetism is the law of electromagnetic induction, discovered by Michael Faraday (1791-   )

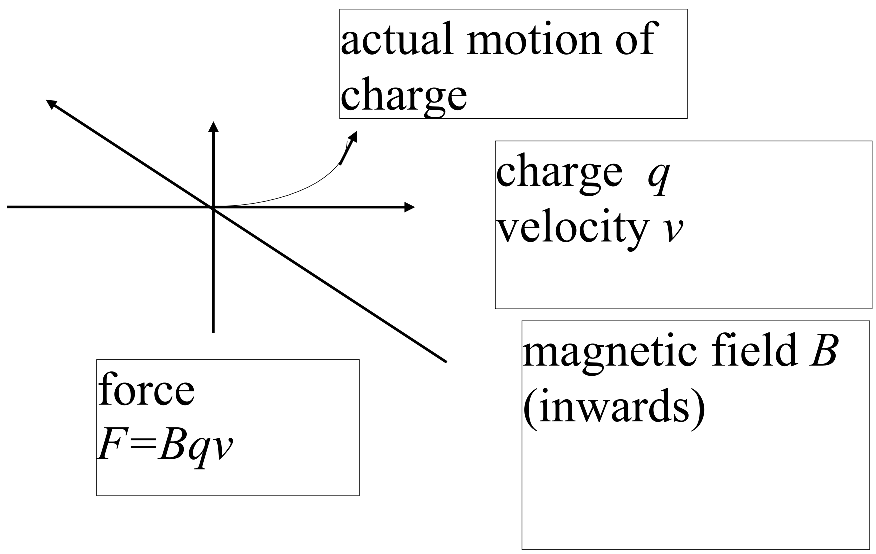
We have seen that:



so a magnetic field should be able to generate an electric field.

We can already see how this should be: a magnetic field can exert a force on a charge, and if this charge moves from one place to another it will change the electric field. The problem is that the charge has to be moving for the magnetic field to exert a force on it.

charge collects up here. Thus, an electric field (pointing downwards) is generated.



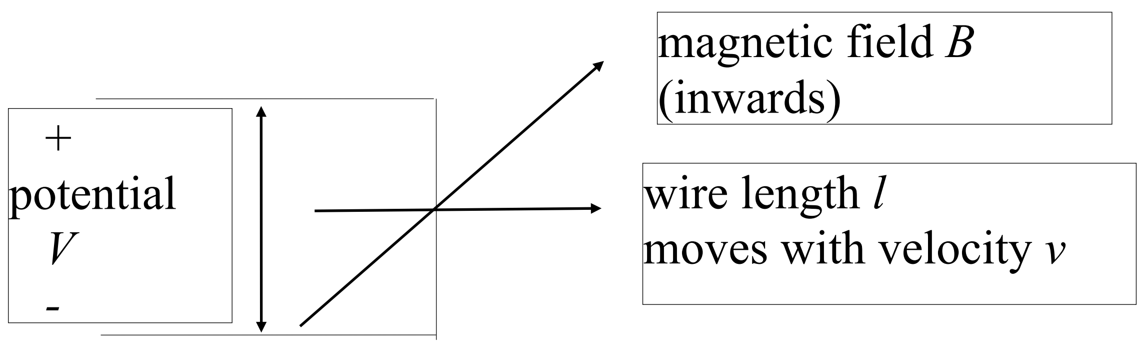
The induced field *opposes* the magnetic movement of the charge. This is known as Lenz's Law and is necessary because of conservation of energy: if the electric force aids the magnetic force, the charge would continue accelerating indefinitely.

In actual fact, at equilibrium the charges carry on moving in their original direction because the electric field builds up until there is no resultant force on the charge:

$$Bqv + Eq = 0$$

The phenomenon with the geometry shown above is actually observed and is known as the *Hall effect*. However it is very small in most materials.

Historically, induction was first demonstrated by physically moving a wire relative to a magnetic field. The mobile charges within the wire are forced to move along with it and therefore experience a magnetic force. A potential difference therefore exists between the ends of the wire, which causes a current to flow and this can be measured.



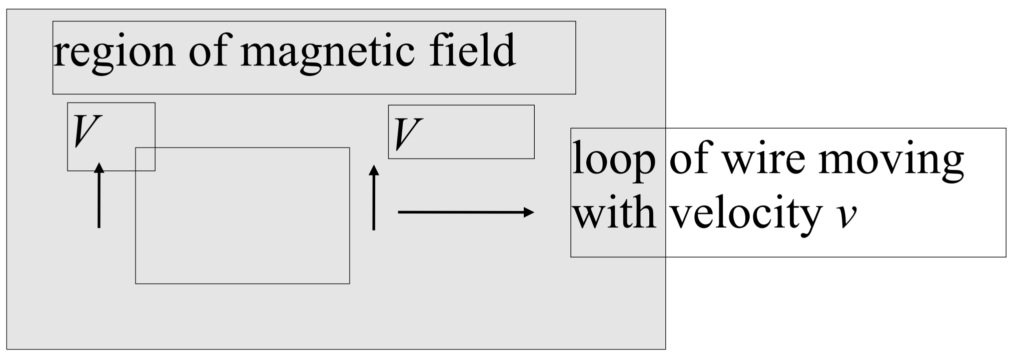
Again, note the perpendicular arrangement of the various quantities. It is easy to show from the above equation that:

$$V = El = -Blv$$

This is known as the *generator effect.* The negative sign automatically incorporates the effect of Lenz's Law.

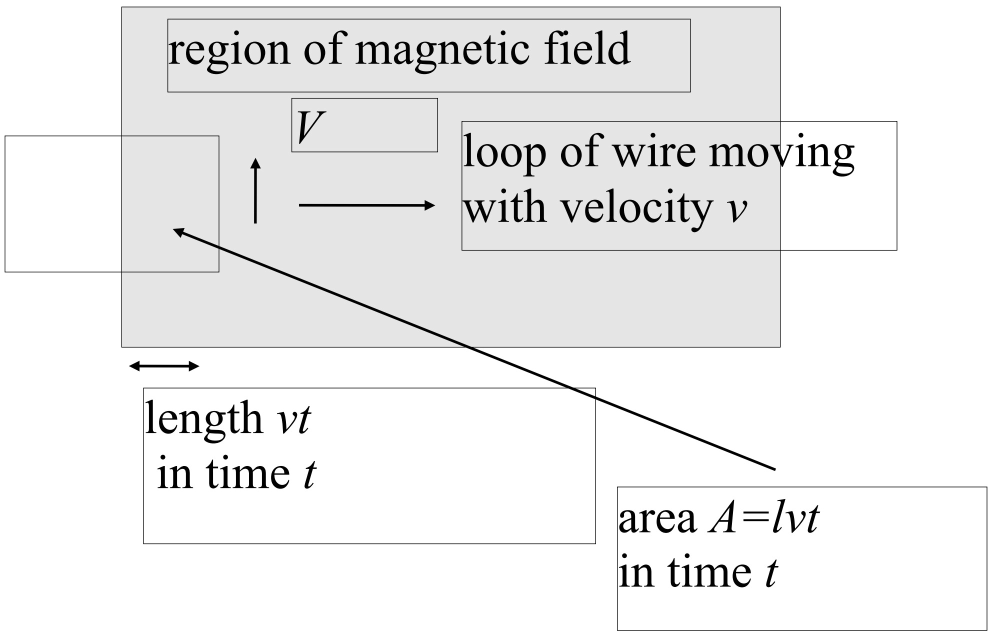
10. Faraday's Law of electromagnetic induction

A wire moving in a magnetic field will have a potential difference between its ends, but this effect is not immediately useful. This is because charges will only respond to a potential difference, and any circuit immersed in the field will have a zero net potential ($\oint \mathbf{E}.d\mathbf{l} = 0$)



On the other hand, we know that an electric current *can* be generated by waving a magnet in the vicinity of a wire (or vice-versa). We therefore need to look more closely at what is happening.

The cancellation is avoided if only part of the loop moves in the magnetic field:



What is happening is that the *magnetic flux linkage* in the circuit is changing.

$$V = -Blv = -BA/t$$

Taking into account that the same effect is obtained if the current is stationary but the magnetic field varied,

$$V = -\frac{d}{dt}BA = -\frac{d\phi_B}{dt}$$

where f_B is the magnetic flux. This is Faraday's Law.