QUANTUM MECHANICS

1. Historical Outline

2. How it works

3. What it means (?)

1. HISTORICAL OUTLINE

(a) Waves that behave like particles

- The Ultraviolet Catastrophe

- Planck's Law $E = hf$

- The Photoelectric Effect - Einstein's equation

- The Compton Effect - the De Broglie relation $p = h/\lambda$

(b) Particles that behave like waves

- Electron Diffraction

- Heisenberg's Uncertainty Principle $\Delta p \, \Delta x \geq h$
  $\Delta E \, \Delta t \geq h$

- The Bohr atom

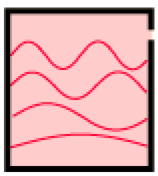
$$\boxed{h = 6.6 \times 10^{-34} \text{ Js}}$$

see http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html
for more details

Blackbody Radiation

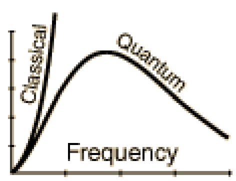
"Blackbody radiation" or "cavity radiation" refers to an object or system which absorbs all radiation incident upon it and re-radiates energy which is characteristic of this radiating system only, not dependent upon the type of radiation which is incident upon it. The radiated energy can be considered to be produced by standing wave or resonant modes of the cavity which is radiating.

Radiation modes in a hot cavity provide a test of quantum theory



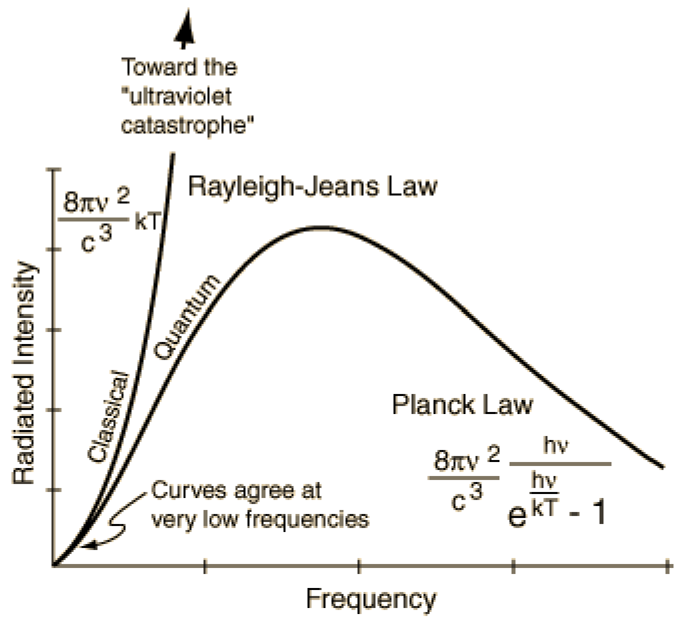
	#Modes per unit frequency per unit volume	Probability of occupying modes	Average energy per mode
CLASSICAL	$\frac{8\pi\nu^2}{c^3}$	Equal for all modes	kT
QUANTUM	$\frac{8\pi\nu^2}{c^3}$	Quantized modes: require $h\nu$ energy to excite upper modes, less probable	$\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$

The amount of radiation emitted in a given frequency range should be proportional to the number of modes in that range. The best of classical physics suggested that all modes had an equal chance of being produced, and that the number of modes went up proportional to the square of the frequency.



But the predicted continual increase in radiated energy with frequency (dubbed the "ultraviolet catastrophe") did not happen. Nature knew better.

Blackbody Intensity as a Function of Frequency



The Rayleigh-Jeans curve agrees with the [Planck radiation formula] for long wavelengths, low frequencies.

The Planck Hypothesis

In order to explain the frequency distribution of radiation from a hot cavity (blackbody radiation) Planck proposed the ad hoc assumption that the radiant energy could exist only in discrete quanta which were proportional to the frequency. This would imply that higher modes would be less populated and avoid the ultraviolet catastrophe of the Rayleigh-Jeans Law.

frequency of radiation, sometimes written as f giving expression E = hf.

$$E = h\nu$$

Quantum energy of a photon.

h = Planck's constant = 6.626×10^{-34} Joule·sec = 4.136×10^{-15} eV·s

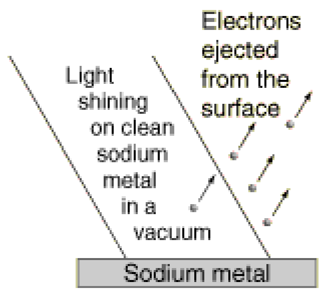
The quantum idea was soon seized to explain the photoelectric effect, became part of the Bohr theory of discrete atomic spectra, and quickly became part of the foundation of modern quantum theory.

Photons: The Quanta of Light

According to the Planck hypothesis, all electromagnetic radiation is quantized and occurs in finite "bundles" of energy which we call photons. The quantum of energy for a photon is not Planck's constant **h** itself, but the product of **h** and the frequency. The quantization implies that a photon of blue light of given frequency or wavelength will always have the same size quantum of energy. For example, a photon of blue light of wavelength 450 nm will always have 2.76 eV of energy. It occurs in quantized chunks of 2.76 eV, and you can't have half a photon of blue light - it always occurs in precisely the same sized energy chunks.

But the frequency available is continuous and has no upper or lower bound, so there is no finite lower limit or upper limit on the possible energy of a photon. On the upper side, there are practical limits because you have limited mechanisms for creating really high energy photons. Low energy photons abound, but when you get below radio frequencies, the photon energies are so tiny compared to room temperature thermal energy that you really never see them as distinct quantized entities - they are swamped in the background. Another way to say it is that in the low frequency limits, things just blend in with the classical treatment of things and a quantum treatment is not necessary.

The Photoelectric Effect



The details of the photoelectric effect were in direct contradiction to the expectations of very well developed classical physics.

The explanation marked one of the major steps toward quantum theory.

The remarkable aspects of the photoelectric effect when it was first observed were:

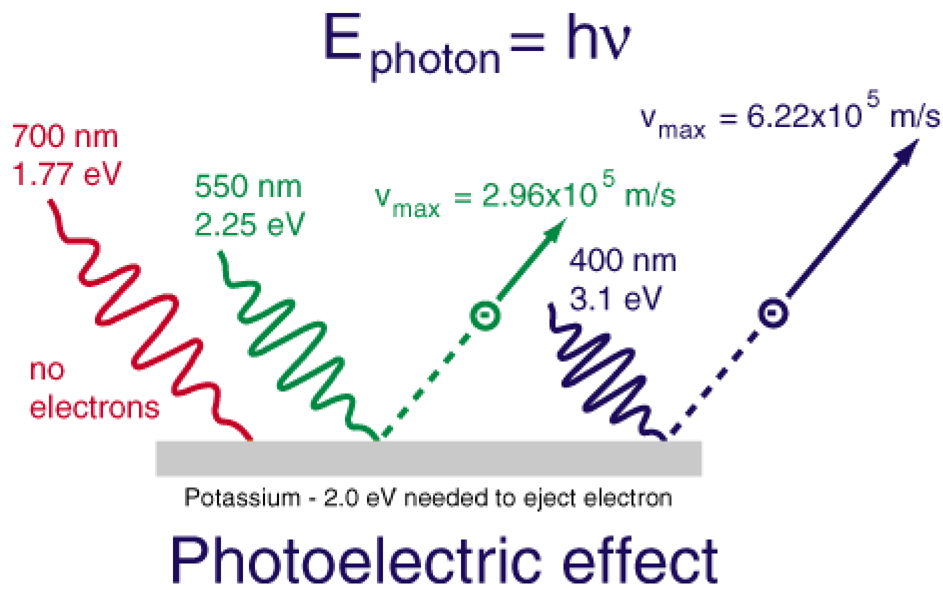
? 1. The electrons were emitted immediately - no time lag!

? 2. Increasing the intensity of the light increased the number of photoelectrons, but not their maximum kinetic energy!

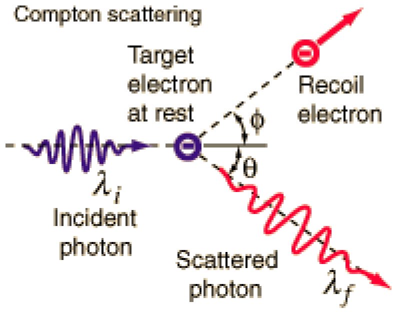
? 3. Red light will not cause the ejection of electrons, no matter what the intensity!

? 4. A weak violet light will eject only a few electrons, but their maximum kinetic energies are greater than those for intense light of longer wavelengths!

Photoelectric Effect

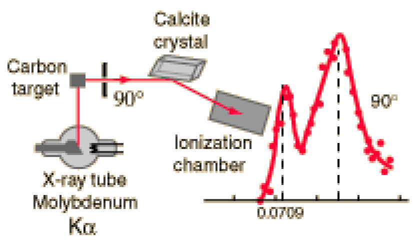


Most commonly observed phenomena with light can be explained by waves. But the photoelectric effect suggested a particle nature for light.



Compton Scattering

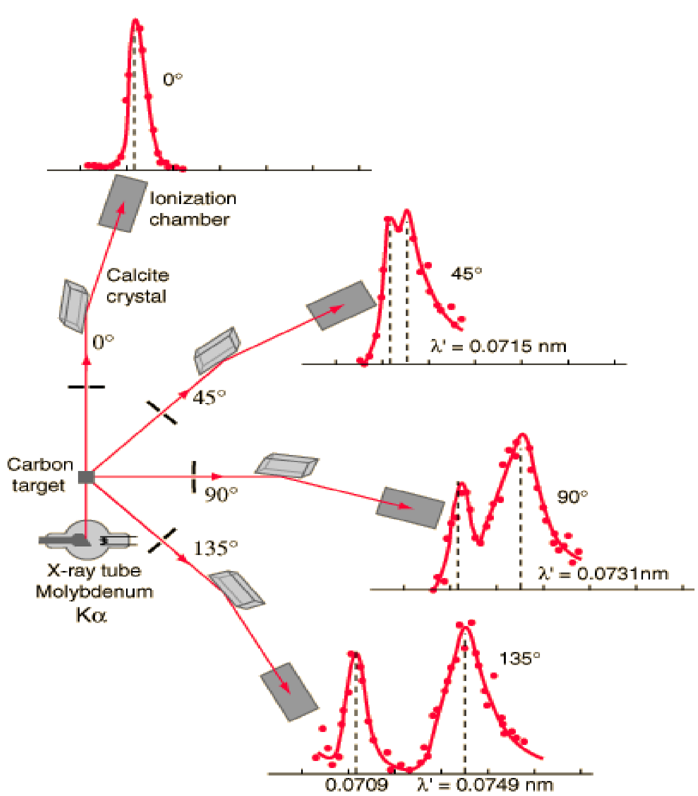
Arthur H. Compton observed the scattering of x-rays from electrons in a carbon target and found scattered x-rays with a longer wavelength than those incident upon the target. The shift of the wavelength increased with scattering angle according to the Compton formula:



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton explained and modeled the data by assuming a particle (photon) nature for light and applying conservation of energy and conservation of momentum to the collision between the photon and the electron. The scattered photon has lower energy and therefore a longer wavelength according to the Planck relationship.

At a time (early 1920's) when the particle (photon) nature of light suggested by the photoelectric effect was still being debated, the Compton experiment gave clear and independent evidence of particle-like behavior. Compton was awarded the Nobel Prize in 1927 for the "discovery of the effect named after him".



DeBroglie Hypothesis

Suggested by De Broglie in about 1923, the path to the wavelength expression for a particle is by analogy to the momentum of a photon. Starting with the Einstein formula:

$$E = mc^2 = KE + m_0c^2$$

Another way of expressing this is $E = \sqrt{p^2c^2 + m_0^2c^4}$

Therefore, for a particle of zero rest mass $p = \dfrac{E}{c}$

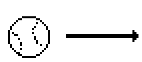
For a photon: $E = h\upsilon = \dfrac{hc}{\lambda}$ *so* $p = \dfrac{hc}{c\lambda} = \dfrac{h}{\lambda}$

DeBroglie Wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

The momentum-wavelength relationship for a photon can then be derived and this DeBroglie wavelength relationship applies to other particles as well.

Does this relationship apply to all particles? Consider a pitched baseball:



v = 40 m/s = 90 mi/hr

m=0.15 kg

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J s}}{(0.15\ \text{kg})(40\ \text{m/s})} = 1.1 \times 10^{-34}\ \text{m}$$

10^{-10}m Atomic diameter
10^{-14}m Nuclear Diameter

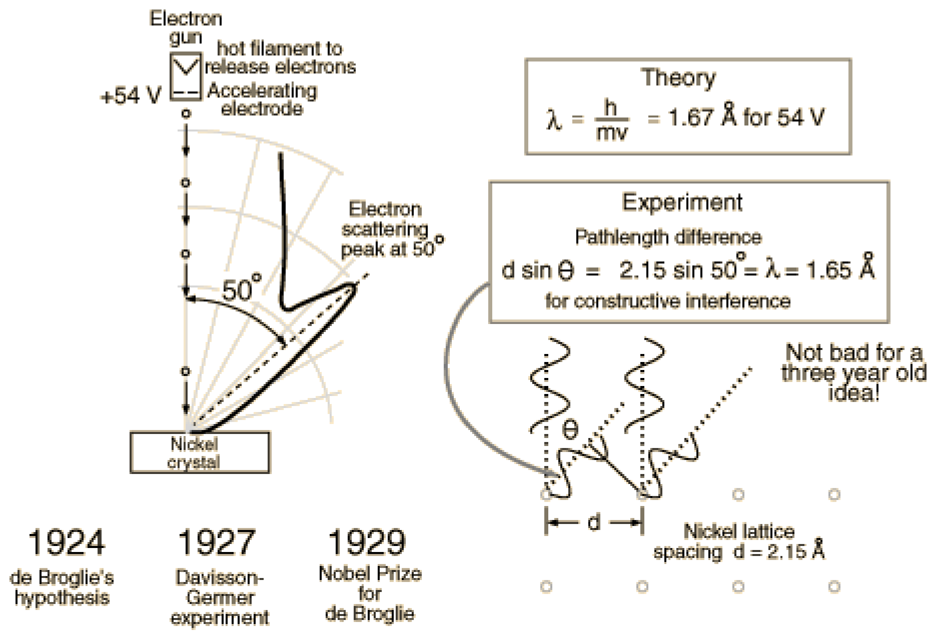
For an electron accelerated through 100 Volts: $v = 5.9 \times 10^6$ m/s

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{Js}}{(9.11 \times 10^{-31}\ \text{kg})(5.9 \times 10^6\ \text{m/s})} = 1.2 \times 10^{-10} = 0.12\ \text{nm}$$

This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.

Davisson-Germer Experiment

(electron diffraction)



This experiment demonstrated the wave nature of the electron, confirming the earlier hypothesis of deBroglie. Putting wave-particle duality on a firm experimental footing, it represented a major step forward in the development of quantum mechanics. The Bragg law for diffraction had been applied to x-ray diffraction, but this was the first application to particle waves.

Note: To date (2004) a similar experiments have been performed showing wave behaviour for atoms, small molecules, and even C_{60} fullerene molecules!

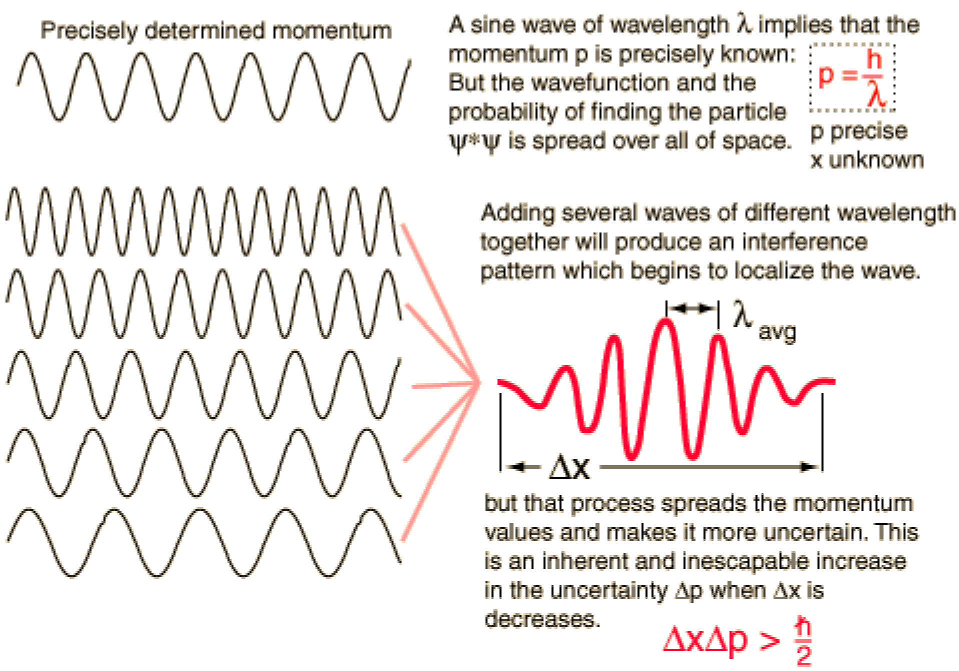
The Uncertainty Principle

The position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision. There is a minimum for the product of the uncertainties of these two measurements. There is likewise a minimum for the product of the uncertainties of the energy and time.

$$\Delta x \Delta p > \frac{\hbar}{2}$$

$$\Delta E \Delta t > \frac{\hbar}{2}$$

This is not a statement about the inaccuracy of measurement instruments, nor a reflection on the quality of experimental methods; it arises from the wave properties inherent in the quantum mechanical description of nature. Even with perfect instruments and technique, the uncertainty is inherent in the nature of things.



Bohr Orbit

Combining the energy of the classical electron orbit with the quantization of angular momentum, the Bohr approach yields expressions for the electron orbit radii and energies:

$$\frac{mv^2}{2} = \frac{(mvr)^2}{2mr^2} = \frac{n^2h^2}{8\pi^2 mr^2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

kinetic energy of electron

expressed in terms of angular momentum

use quantization of angular momentum

set equal to total energy of classical orbit

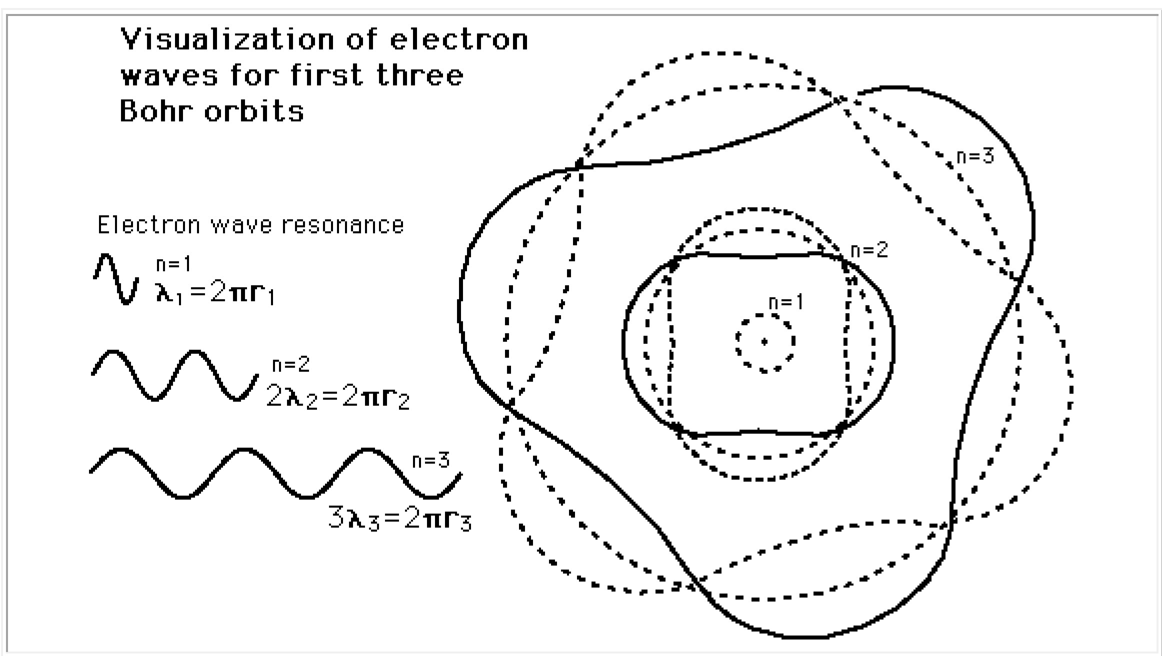
This is for hydrogenic atoms; the use of the atomic number Z is appropriate only if there is only one electron.

Substitution for r gives the Bohr energies and radii:

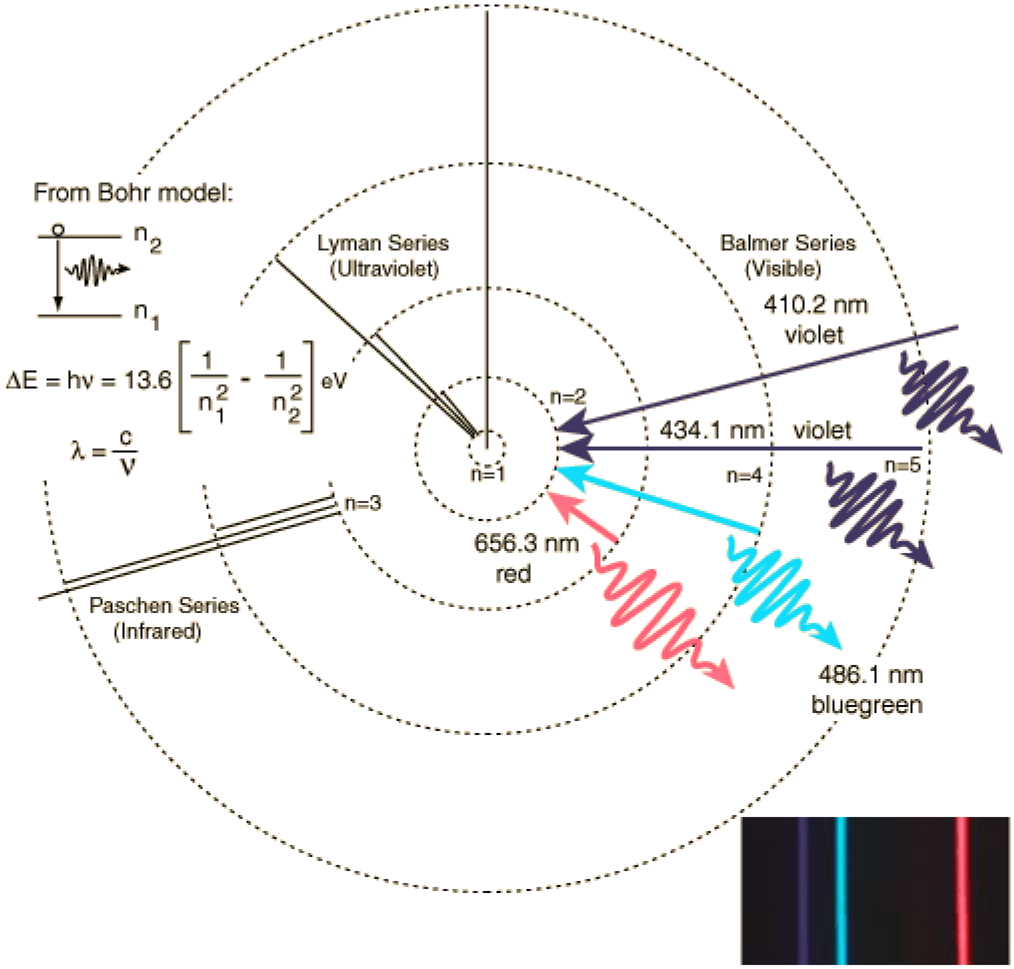
$$E = -\frac{Z^2 me^4}{8n^2h^2\varepsilon_0^2} = -\frac{13.6Z^2}{n^2}\ \mathrm{eV}$$

$$r = \frac{n^2h^2\varepsilon_0}{Z\pi me^2} = \frac{n^2 a_0}{Z}$$

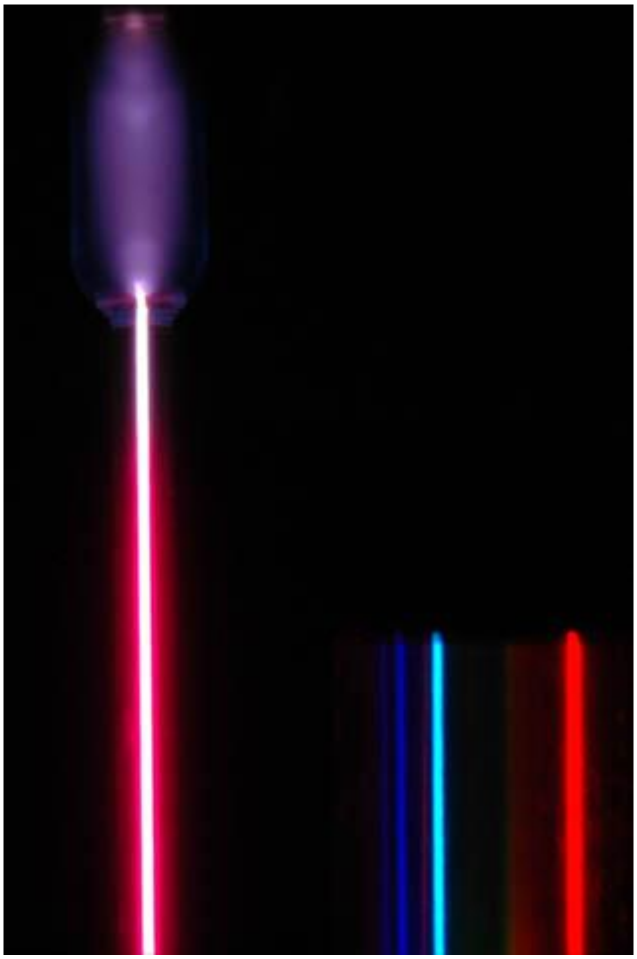
$a_0 = 0.529$ Å = Bohr radius



Hydrogen Spectrum



This spectrum was produced by exciting a glass tube of hydrogen gas with about 5000 volts from a transformer. It was viewed through a diffraction grating with 600 lines/mm. The colors cannot be expected to be accurate because of differences in display devices.



At left is a hydrogen spectral tube excited by a 5000 volt transformer. The three prominent hydrogen lines are shown at the right of the image through a 600 lines/mm diffraction grating.

An approximate classification of spectral colors:

- Violet (380-435nm)
- Blue(435-500 nm)
- Cyan (500-520 nm)
- Green (520-565 nm)
- Yellow (565- 590 nm)
- Orange (590-625 nm)
- Red (625-740 nm)

Radiation of all the types in the electromagnetic spectrum can come from the atoms of different elements. A rough classification of some of the types of radiation by wavelength is:

- Infrared > 750 nm
- Visible 400 - 750 nm
- Ultraviolet 10-400 nm
- Xrays < 10 nm