# Statistics in Cosmology

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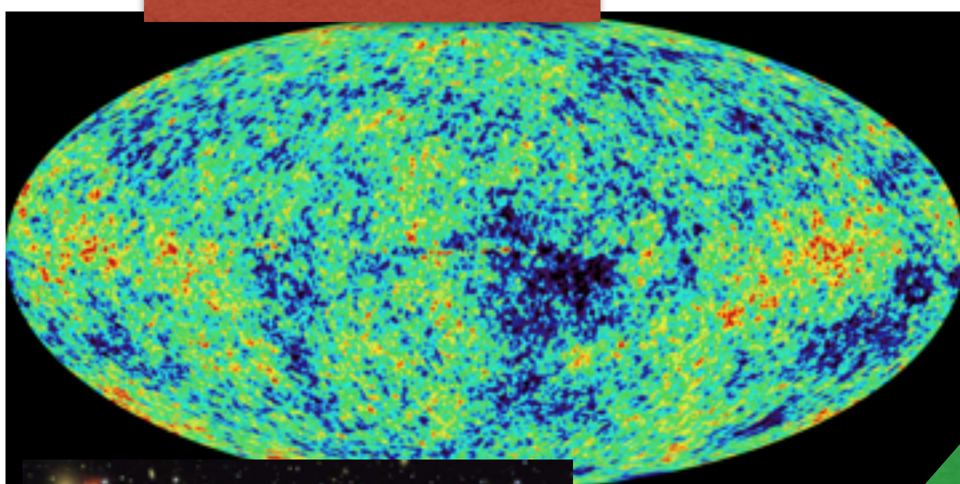
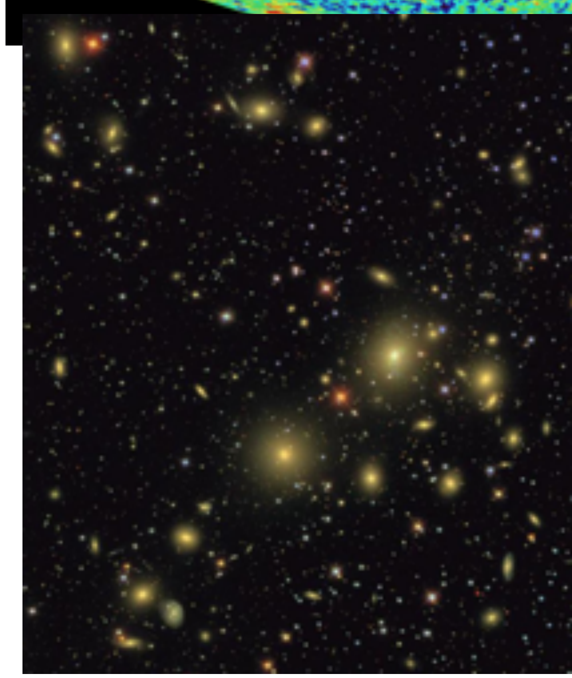
Cosmos in the Med

26 September 2014

# What are we actually trying to do?

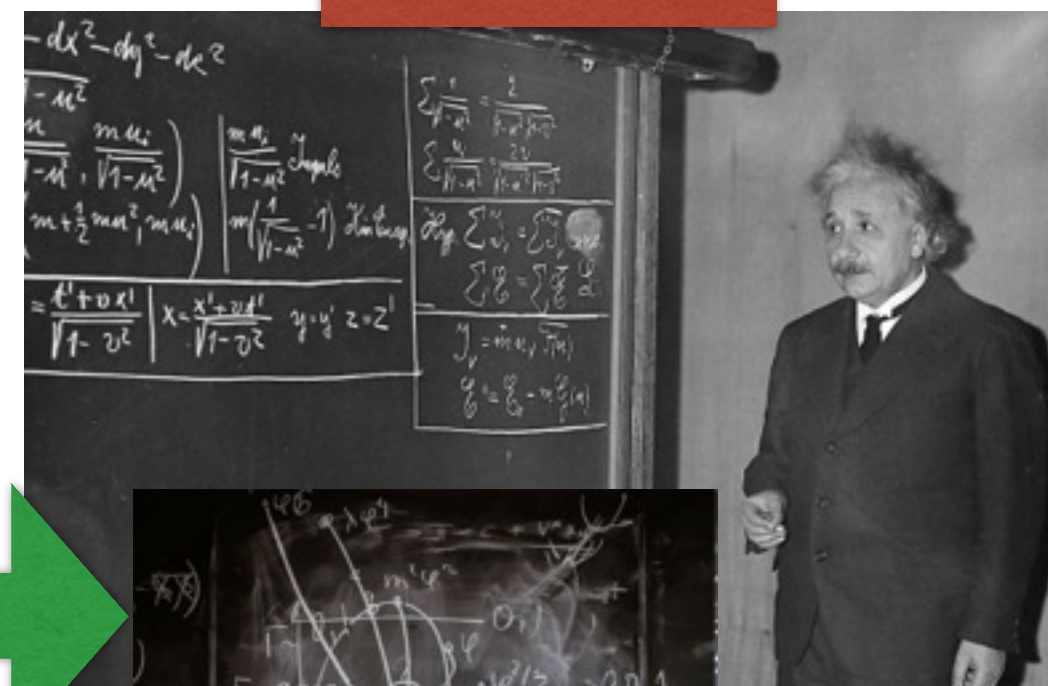
DATA





measurement errors, uncertainty in the theory

THEORY



What came first?
Data or theory?

Can we have 100% measurement precision?

**NO**

Can we be 100% sure of our theory?

**NO**

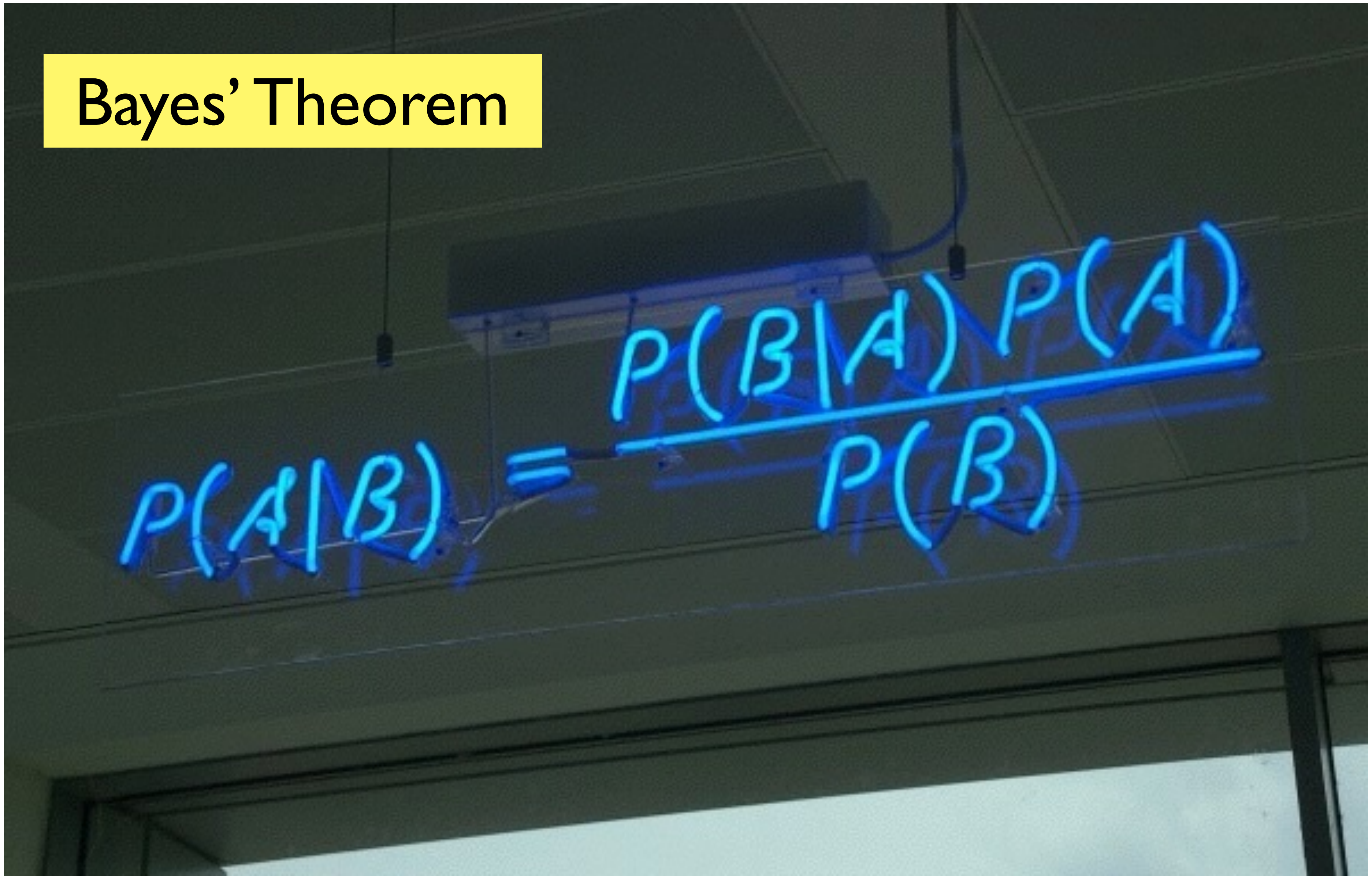
Does that mean there are no true laws of physics?

We have incomplete information about the Universe.
We would like to make sensible decisions about theories.


Truth is your degree of belief. Let probability be your guide.

Statement: A is true (e.g. this is the true value of my measurement) $\rightarrow P(A)$

B is true (e.g. this is the true theory) $\rightarrow P(B)$



**Bayesian inference:**
How can we make sensible decisions based on incomplete information?

**Posterior**
How probable is my theory *AFTER* I measure the data?

**Likelihood**
How probable is my data given the model?

**Prior**
What do I know about my theory *BEFORE* I measure the data?

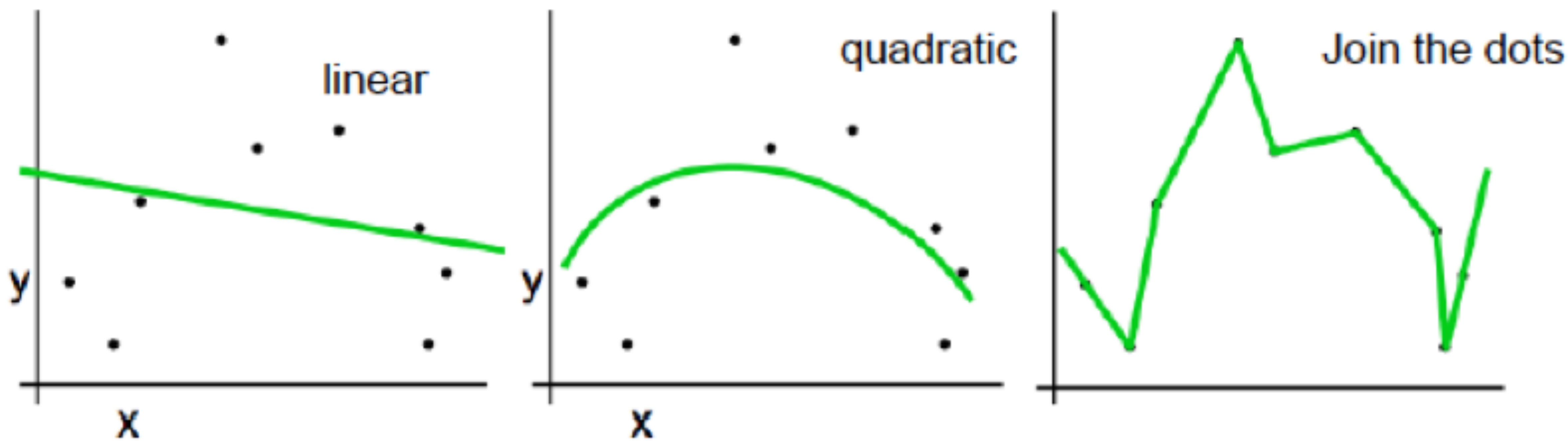
$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

**Evidence**
How probable is my measured data?

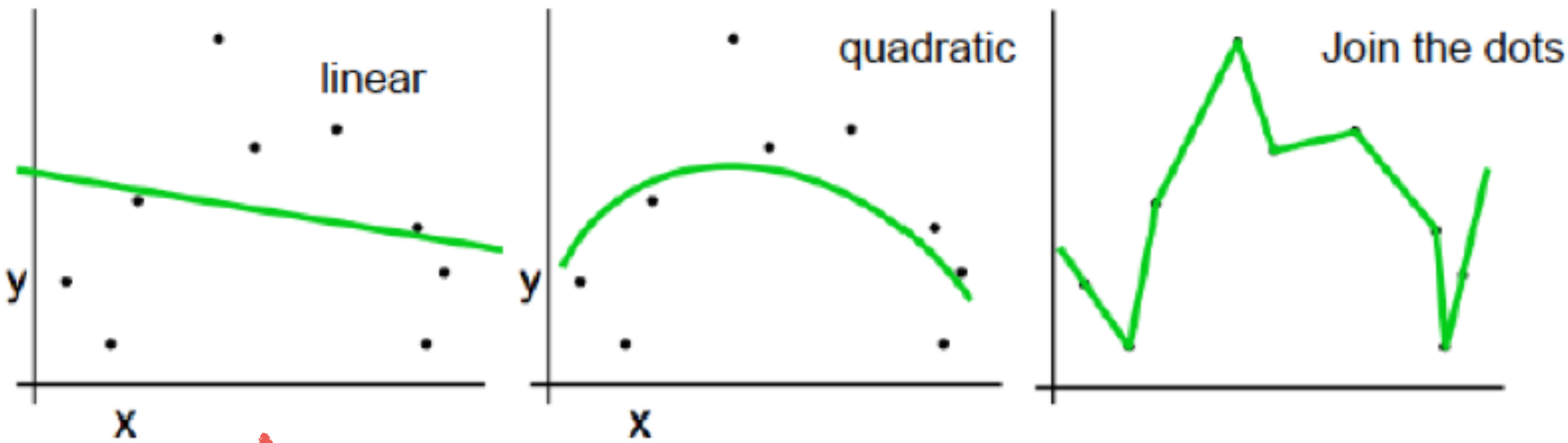
Bayes' Theorem is our most useful tool for going from data to theory and back.

## How to choose a theory after you've collected your data

Which is best?



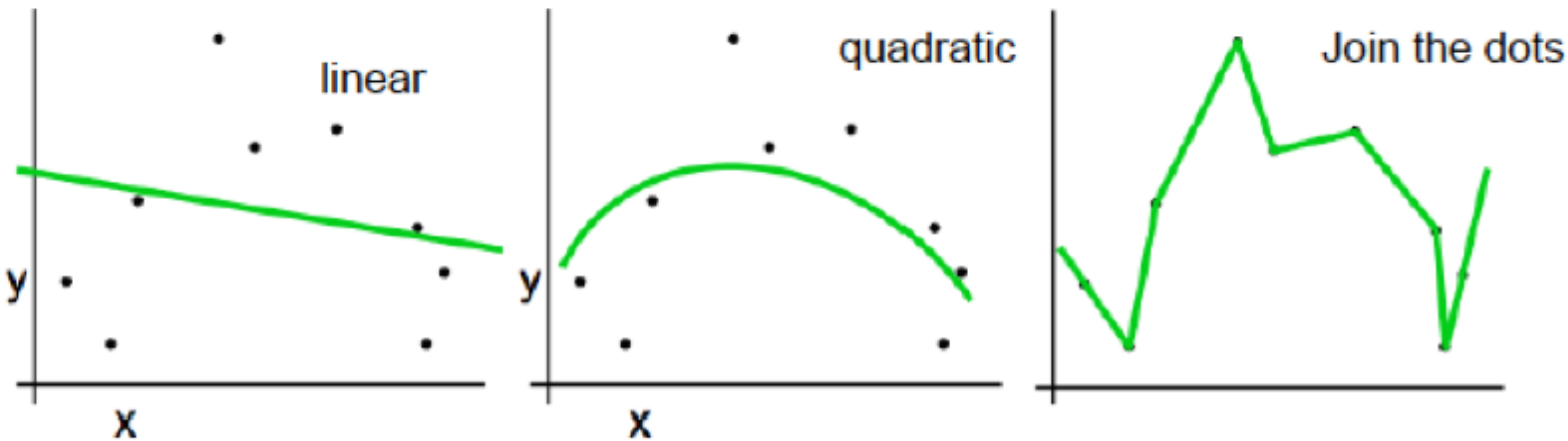
# Which is best?




$$y = A + Bx$$

**2 free parameters: *A, B***

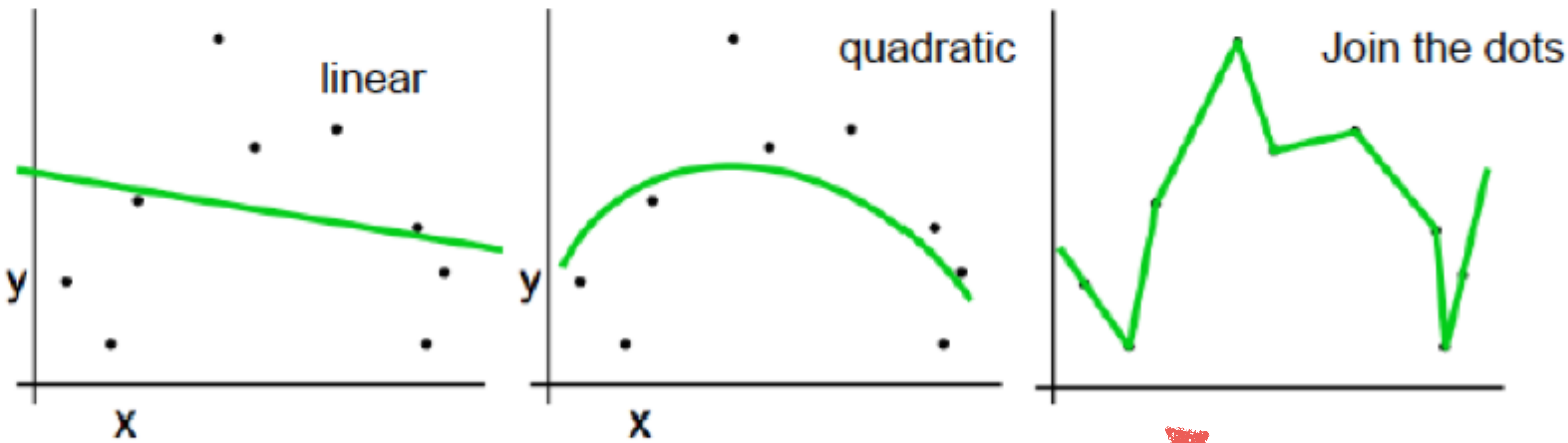
# Which is best?




$$y = A + Bx + Cx^2$$

**3 free parameters: *A, B, C***

# Which is best?

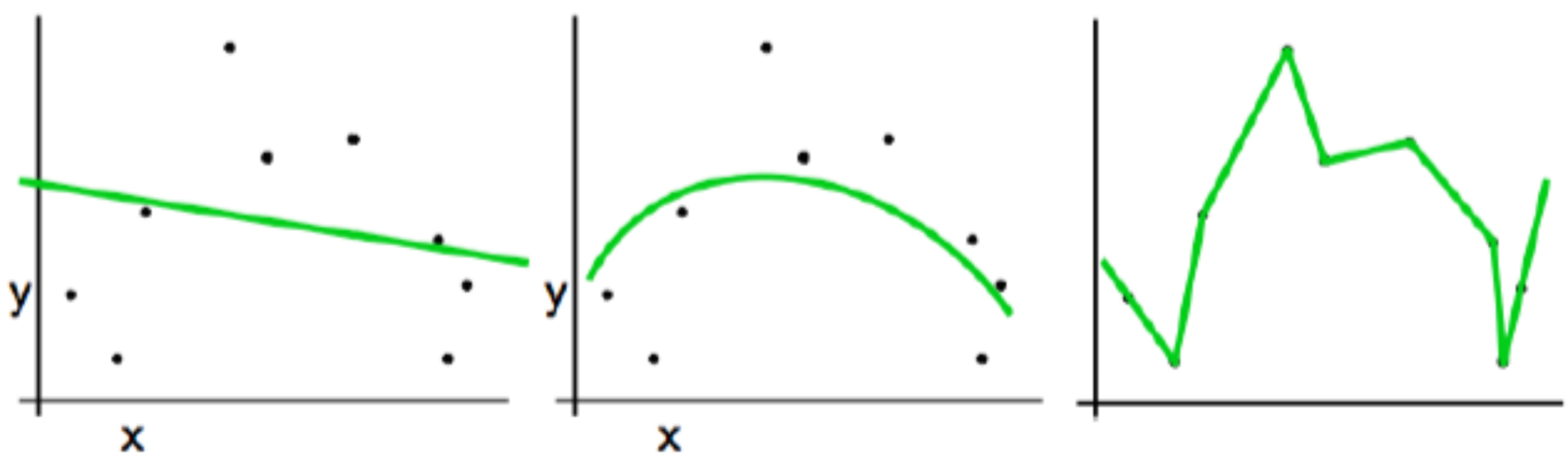



$$y = A + B[x_1, x_2] + C + D[x_3, x_4] + E + F[x_5, x_6] + ...$$

**lots of free parameters: *A, B, C, D, E, F, …***

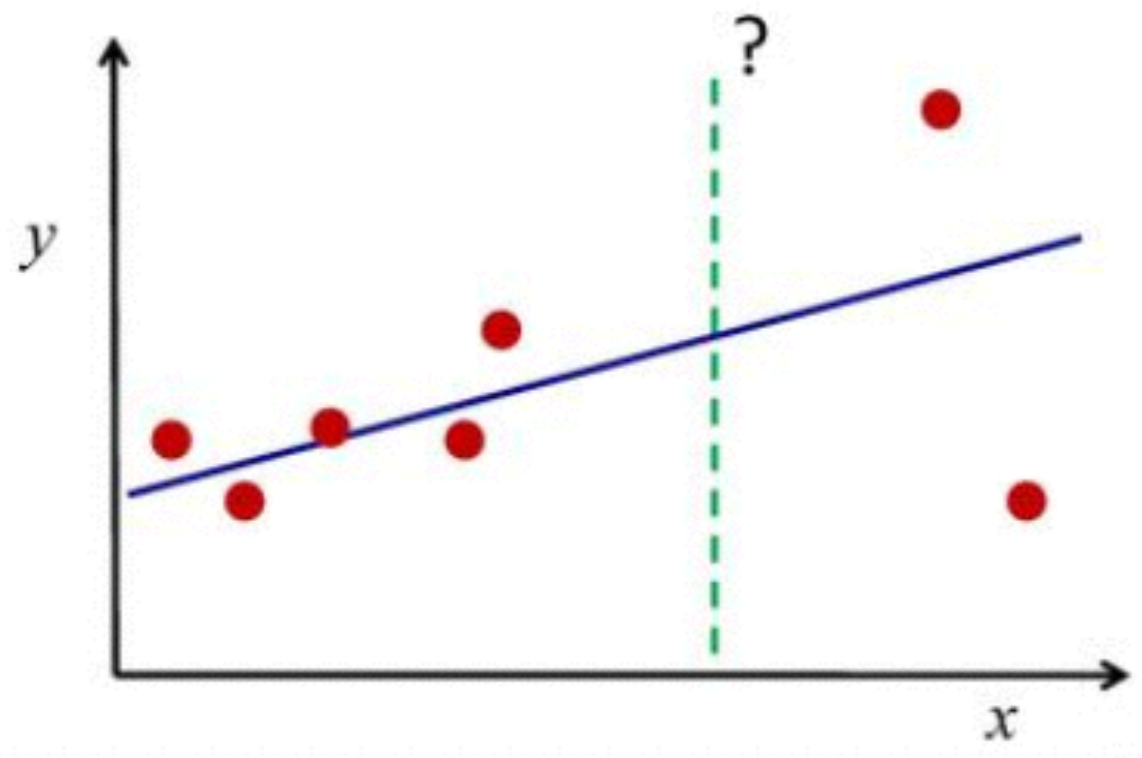
**Why not just choose the model with the best fit to the data?**

# What do we really want?



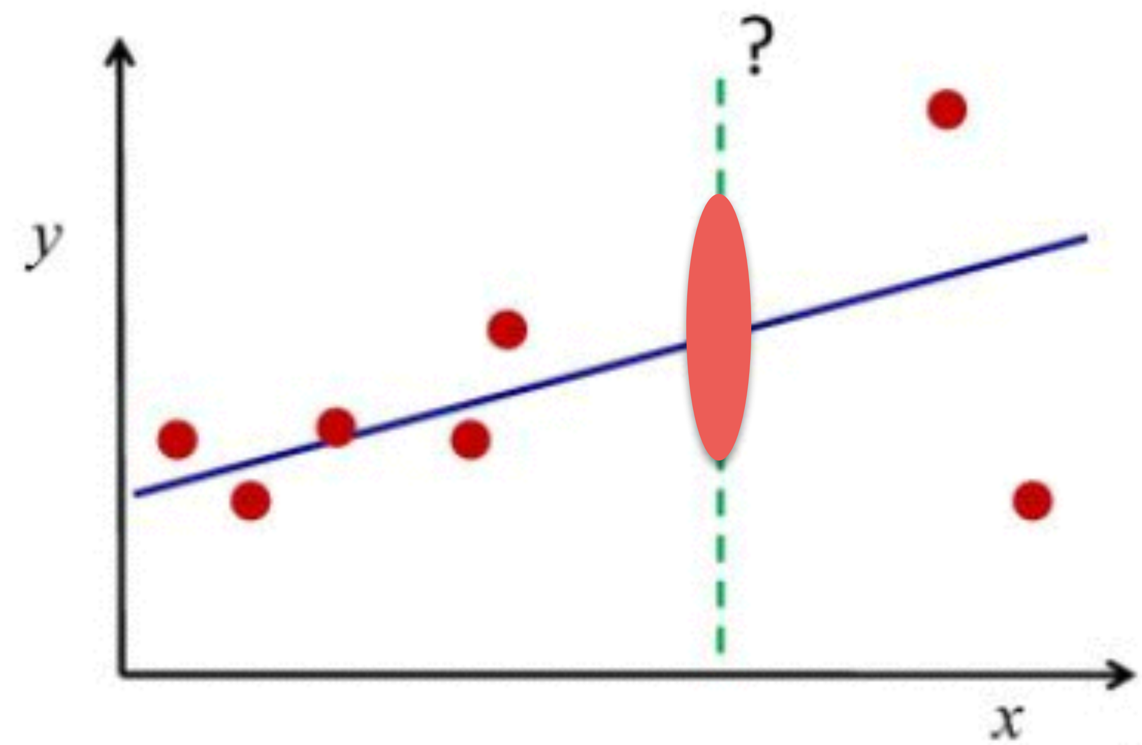

**Why not just choose the model with the best fit to the data?**

**How well are you going to predict future data?**




How well are you going to predict future data?

So which model should we choose?

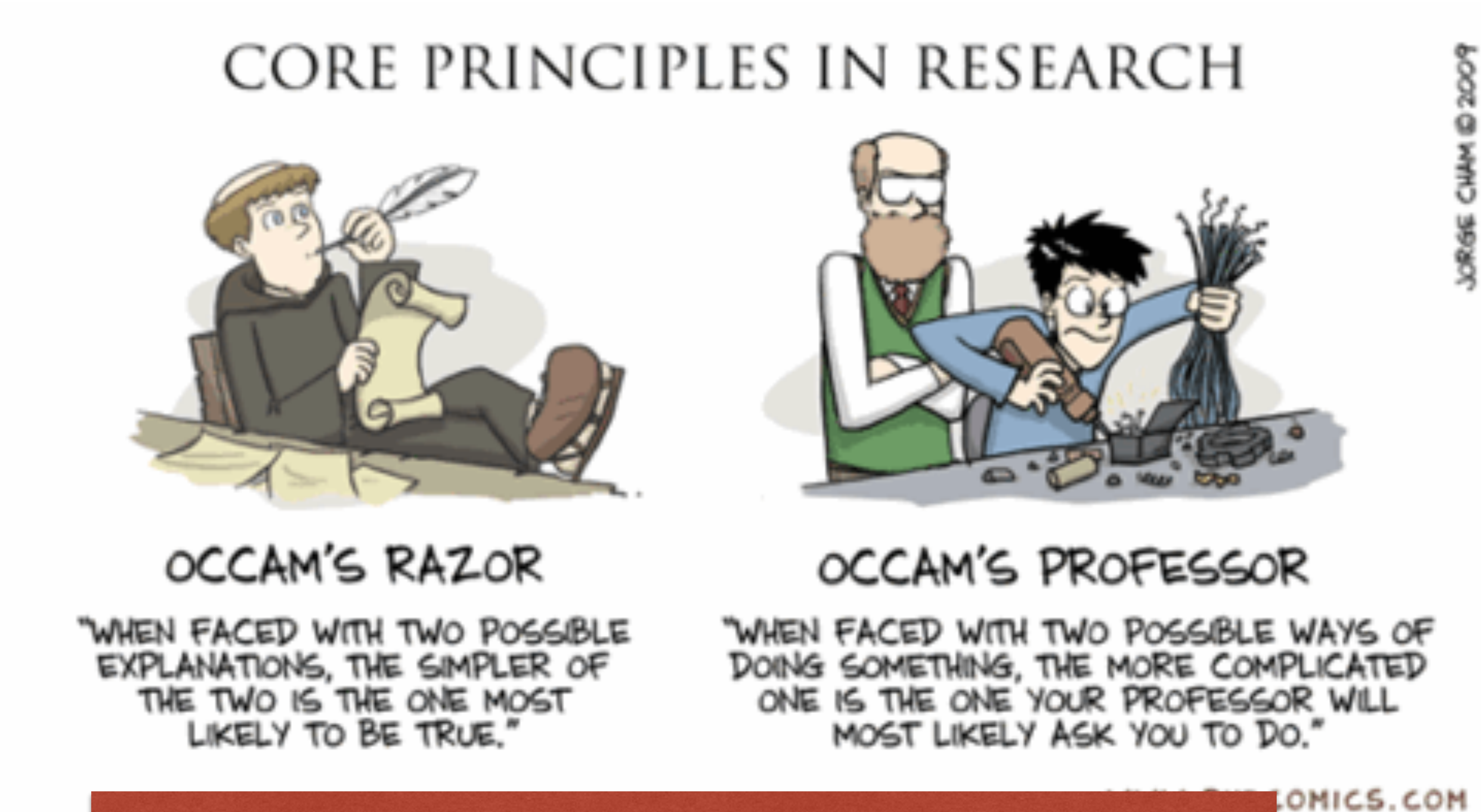



**How well are you going to predict future data?**

**So which model should we choose?**

**Use Occam's razor!** **(i.e. *Keep it simple, stupid.*)**

**Choose the model with the least parameters which fits your data reasonably well.**




When you have chosen your model, then find the values of the parameters.

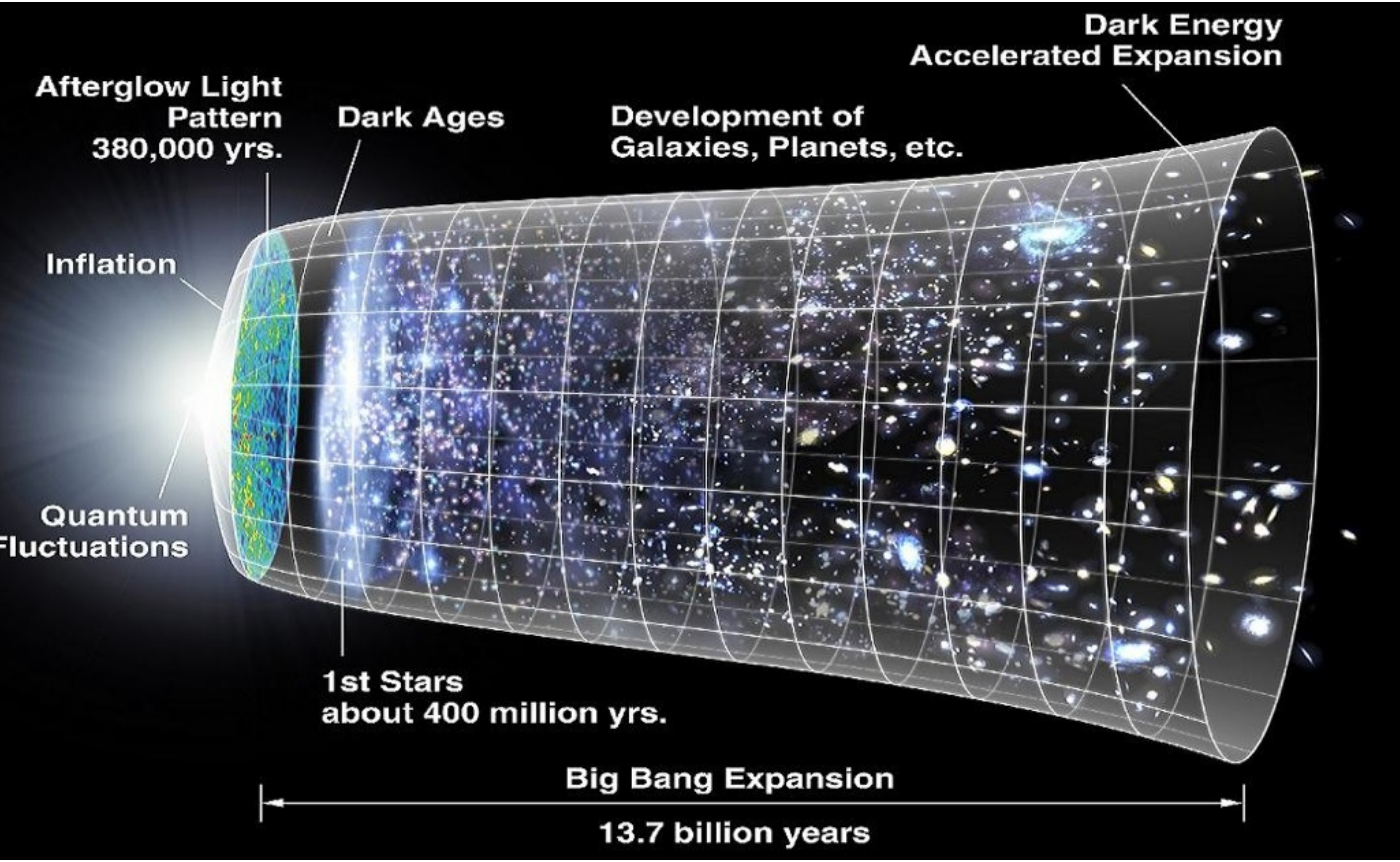
"Assuming a model *M*, what are the values of the parameters?"

## The Concordance Model (where all the data agree)

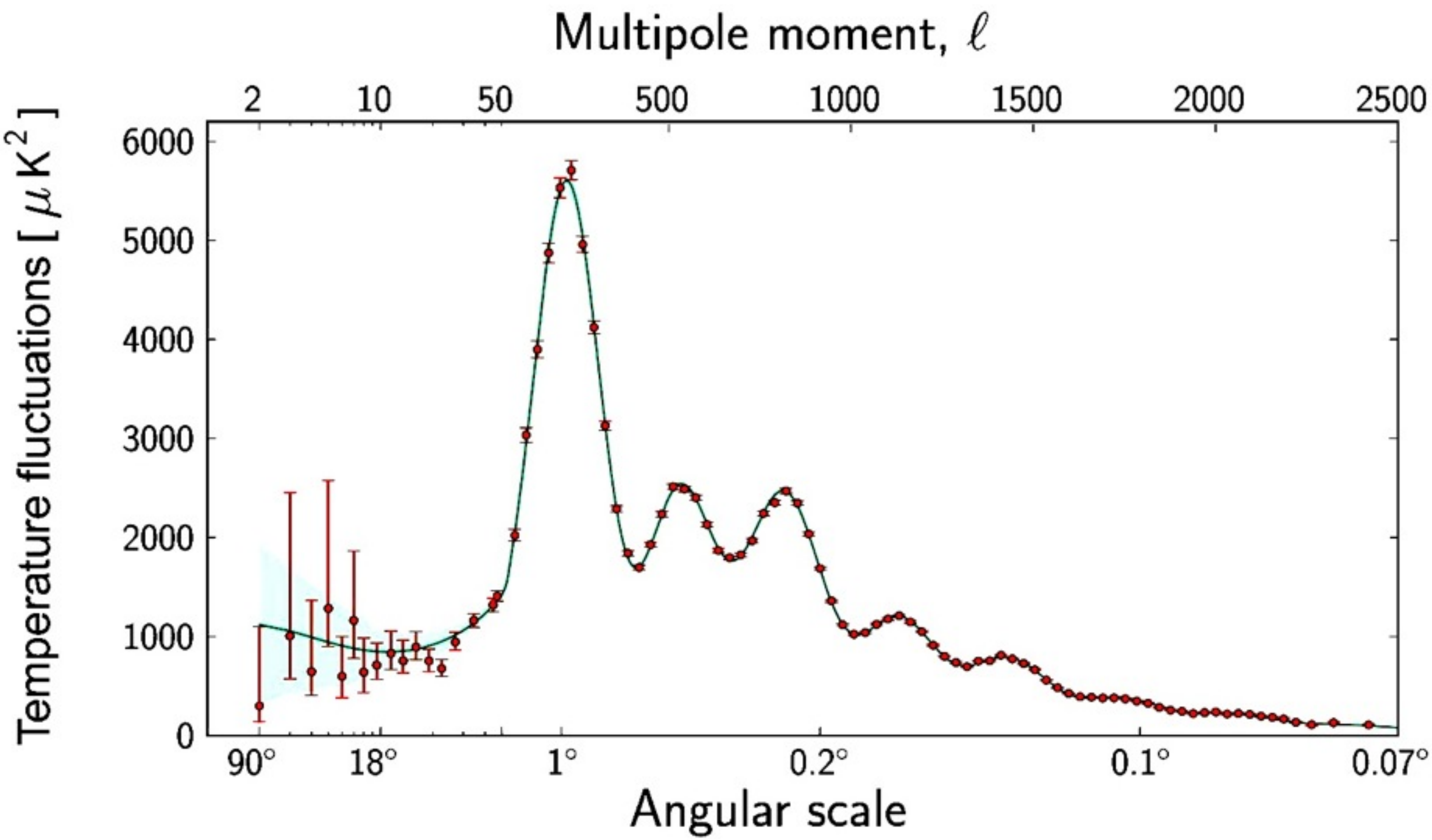
Just 6 parameters:

1. **Hubble constant (expansion of the Universe)**
2. **Amount of ordinary matter**
3. **Amount of dark matter**
4. **Amount of dark energy**
5. **Size of initial fluctuations**
6. **Time when first galaxies formed**

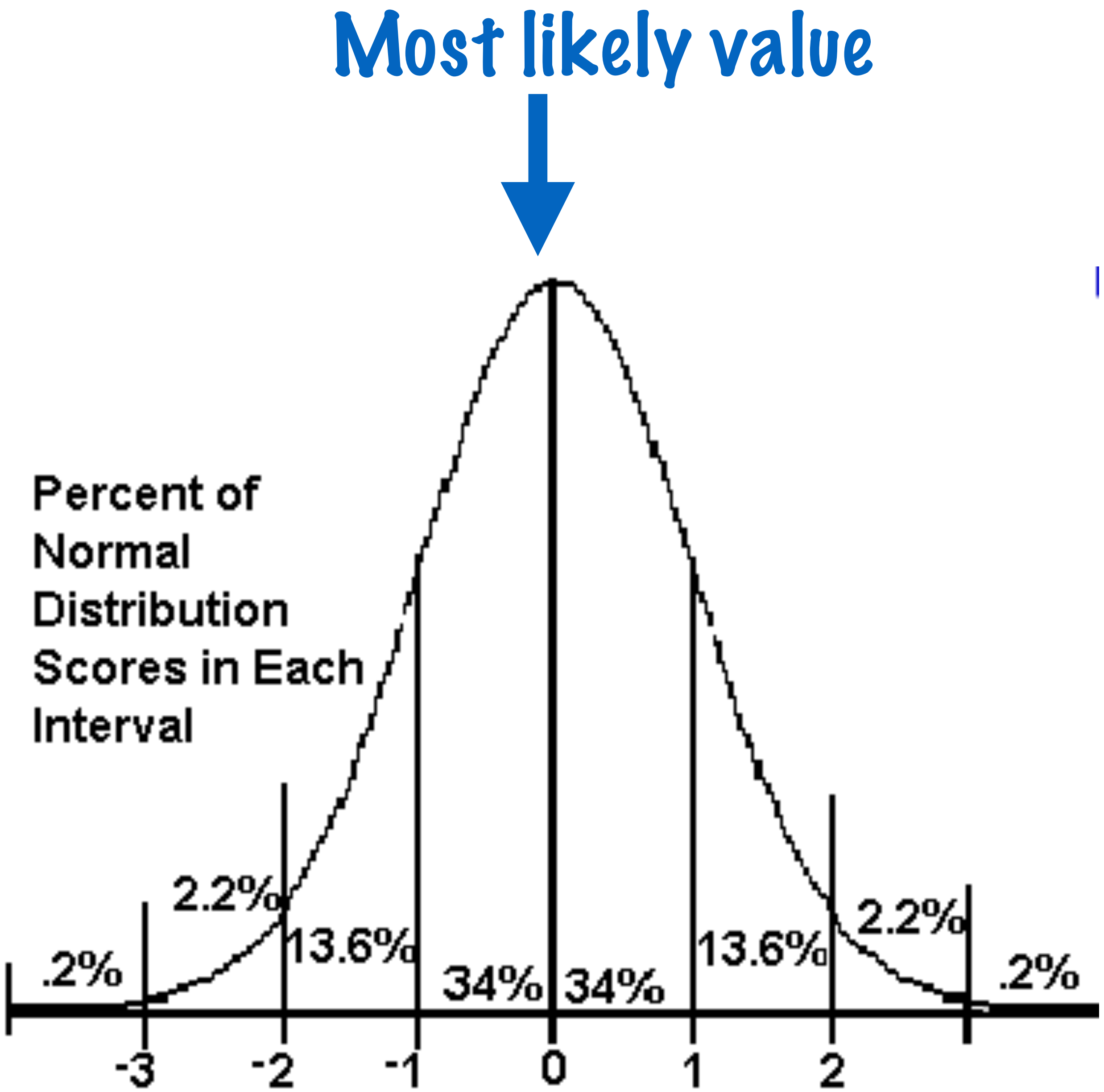
**… and we're done***

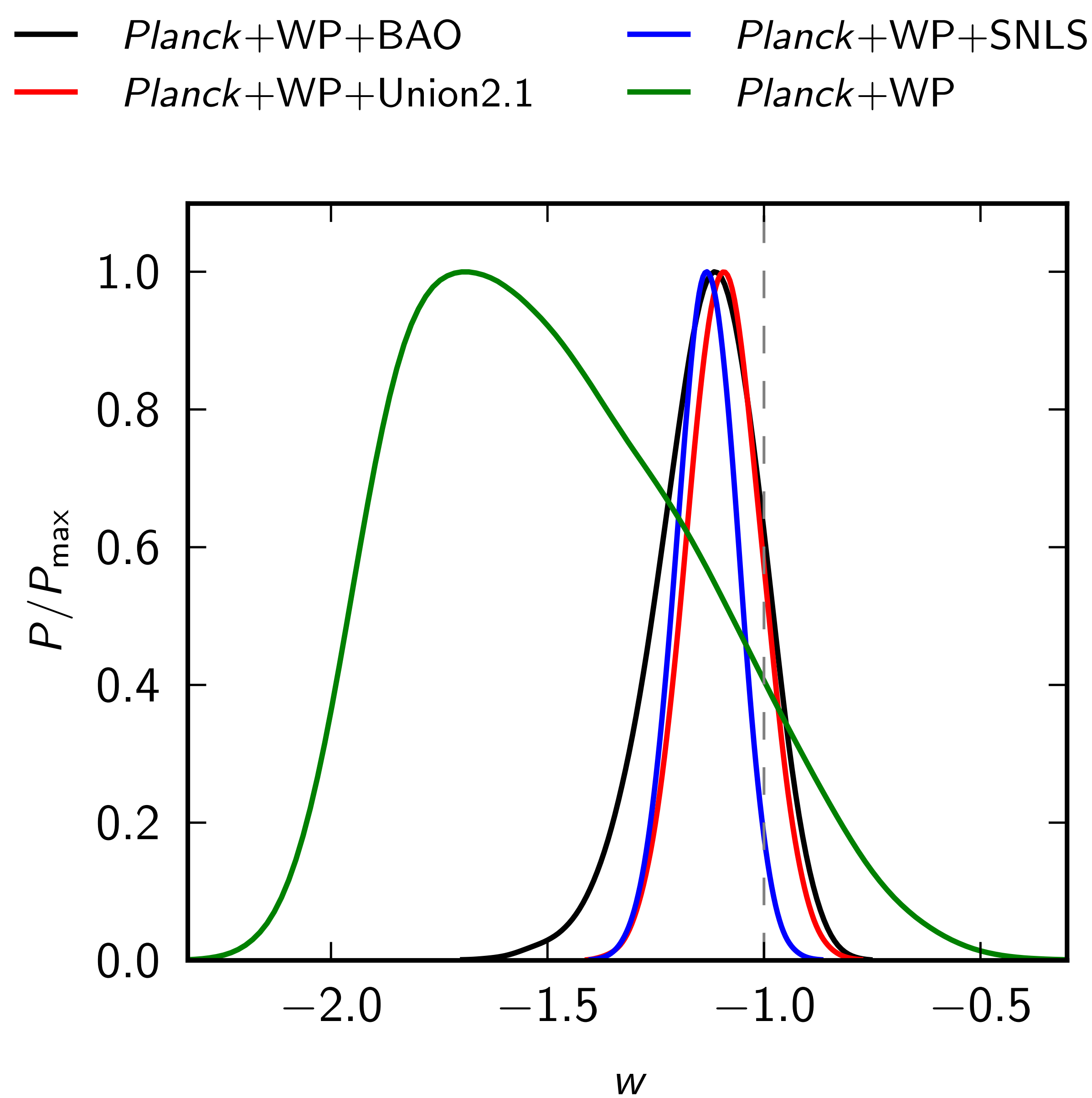

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

# Why we believe the Big Bang theory



# “But I thought you said we cannot have 100% precision.”



Why?

Because infinite experimental precision is impossible.
i.e. **Your parameter values themselves have an error bar.**

| | |
|---|---|
| $H_0$ | $70.4^{+1.3}_{-1.4}$ km s$^{-1}$ Mpc$^{-1}$ |

at 95% confidence.

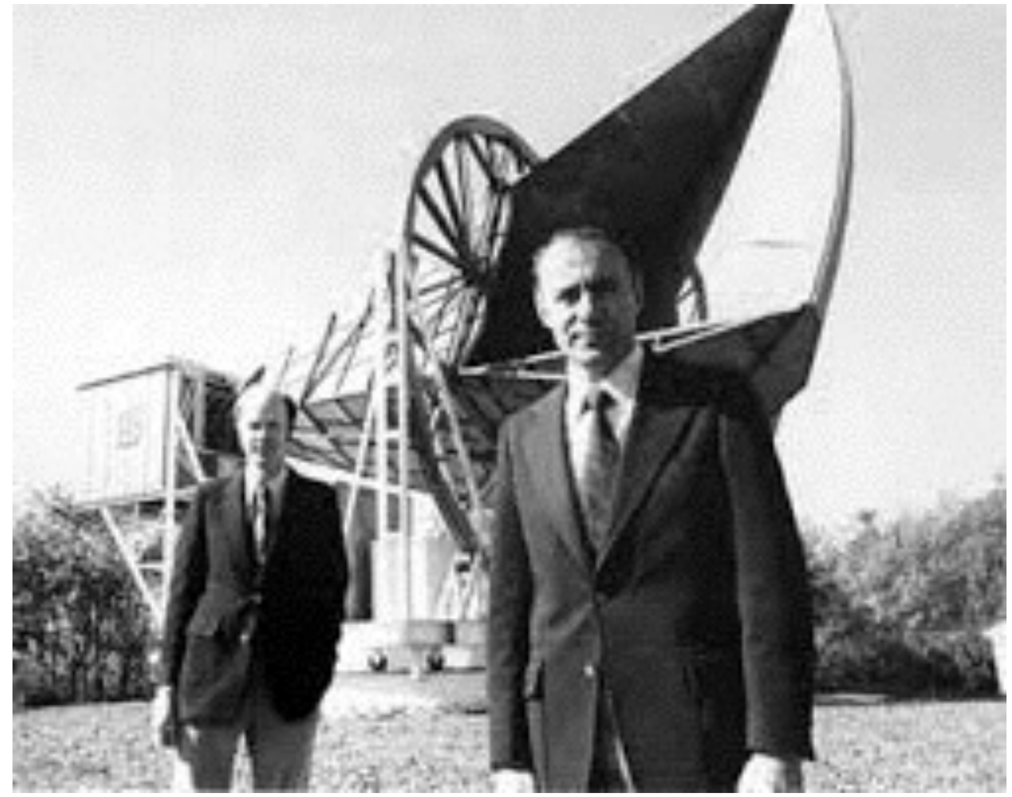
We call this error bar a **confidence region**.

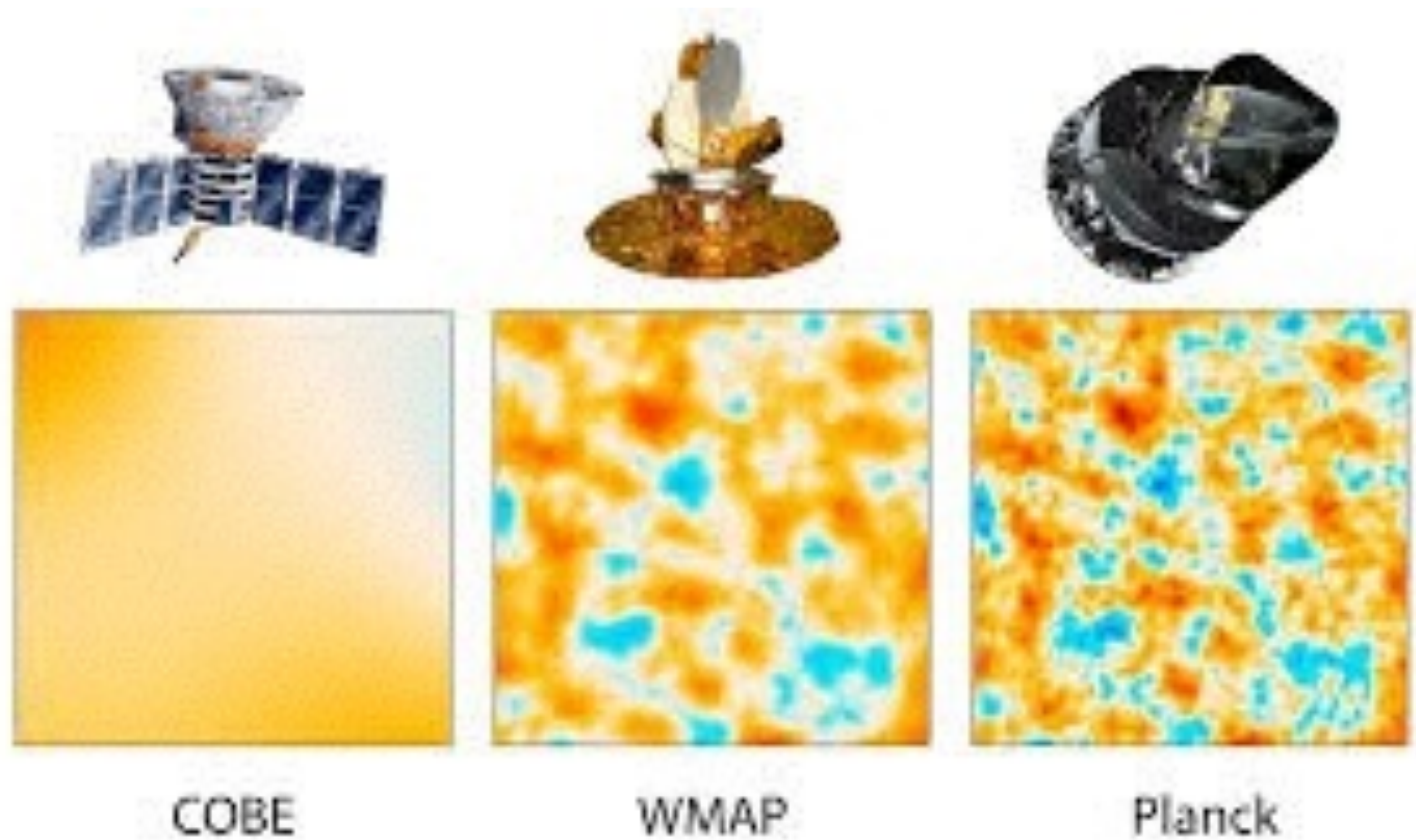
**= the probability of a measuring that value next time I carry out my experiment.**

Different experiments can give different confidence intervals.
They can also give a different position for the most likely value.

Solutions:

- **Make better instruments**
- Collect more data
- Combine the data from different experiments





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**per day**

Solutions:

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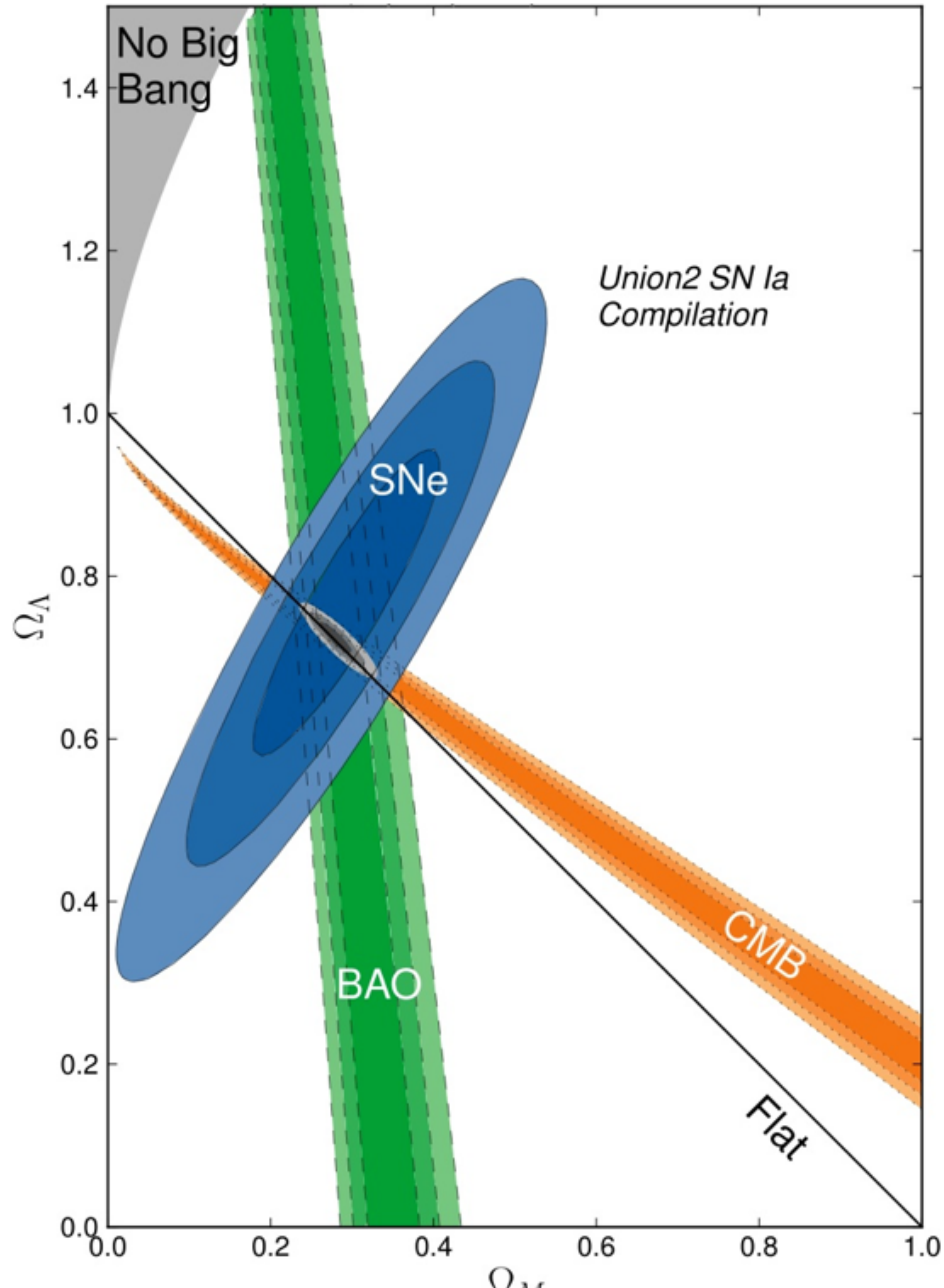
- **Combine the data from different experiments**



# Parameter estimation problems

- What is the value of the Hubble constant today?
- What is the amount of dark matter in the Universe?

The answers we obtain depend on the model we choose…

*We are not quite done: **What happens if there is more than one reasonable model?**

Then we need **model comparison (trickier than parameter estimation)**

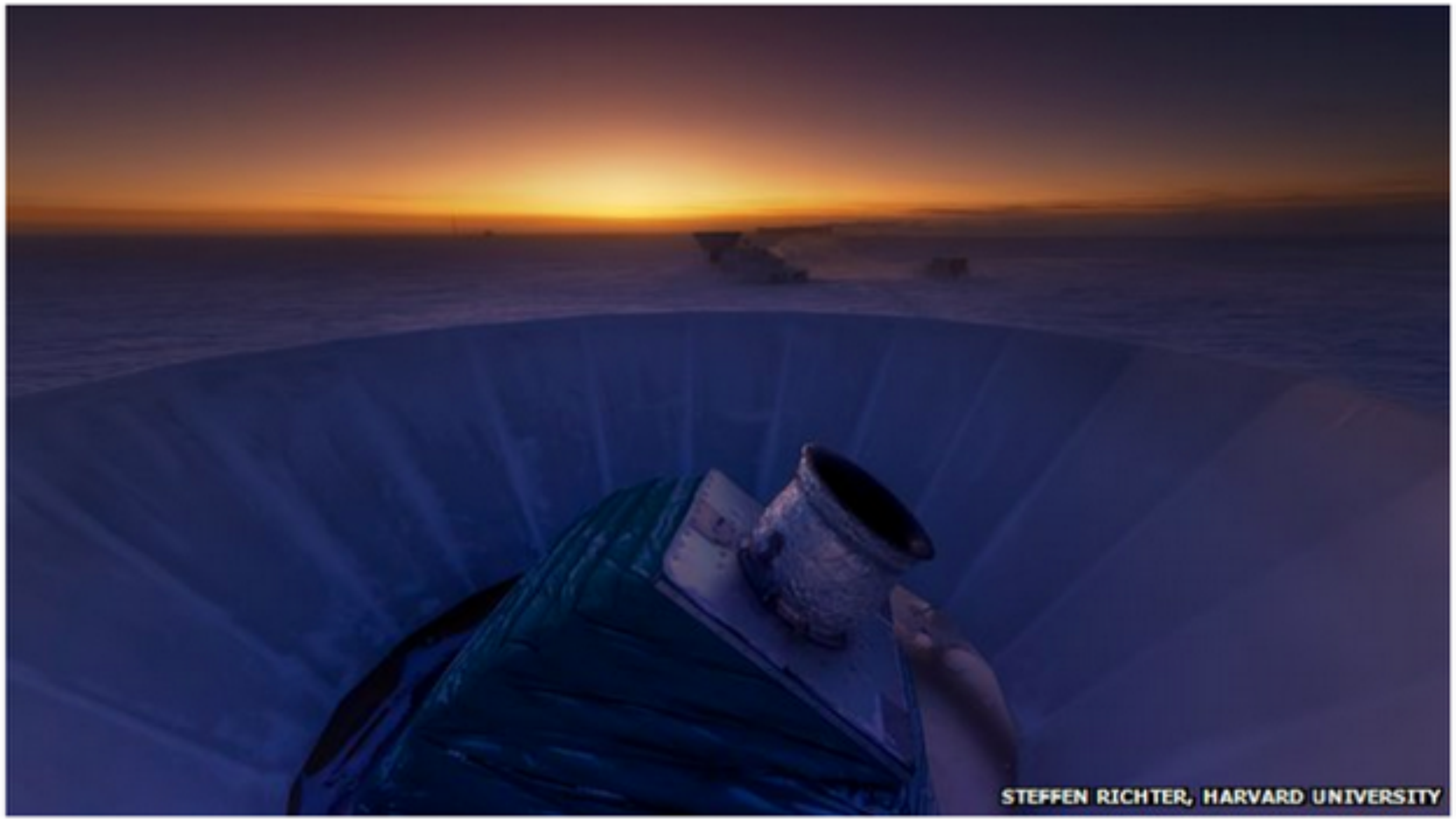
**The interesting questions in cosmology:**

- Is the Universe flat or not?
- Are the initial quantum fluctuations simple or was there a complex mechanism?
- Is dark energy constant or does it change in time?
- Did the very early accelerated expansion of Universe produce gravitational waves?

**17 March 2014**
**“The work will be scrutinised carefully, but already there is talk of a Nobel.”**

# Cosmic inflation: 'Spectacular' discovery hailed

**By Jonathan Amos**
Science correspondent, BBC News



The measurements were taken using the BICEP2 instrument at the South Pole telescope facility

**Scientists say they have extraordinary new evidence to support a Big**

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**23 September 2014**

**"BICEP 2 claimed that 20% of the fluctuations emerging from the big bang were due to gravitation waves… but the Planck Satellite has come out with more data confirming that the limit is less than 10%."**

# Cosmic inflation: BICEP 'underestimated' dust problem

**By Jonathan Amos**
Science correspondent, BBC News


BICEP's South Pole telescope targeted what the team hoped was a relatively clean part of the sky

**One of the biggest scientific claims of the year has received another set-back.**

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"I contacted Steven and said 'let's refine this bet'... it is of course irrelevant who wins the bet."


# Space inflation puts physicists' bet in jeopardy

5 hours ago

One of the biggest scientific claims of the year has received another set-back, putting the result of a bet between two significant physicists in jeopardy.

In March, the Today programme reported on news that the US BICEP team said it had found a pattern on the sky left by the rapid expansion of space just fractions of a second after the Big Bang.

# Lessons from statistics and cosmology

Never throw away your data.
Learn from experience.
Question most theories. Believe some of them.
Know your degree of belief.

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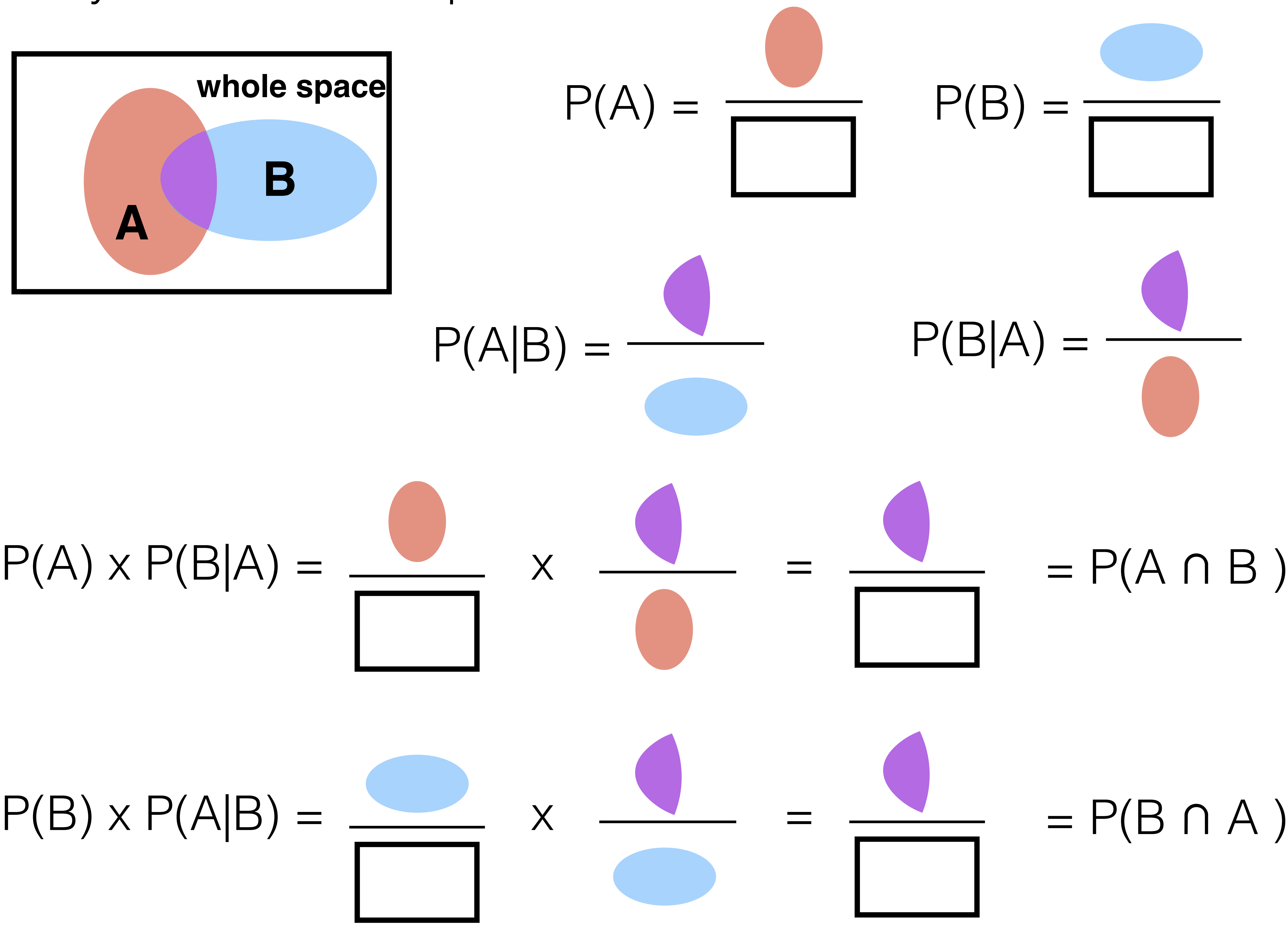
# Bayes' Theorem in pictures



We can see that $P(A) \times P(B|A) = P(B) \times P(A|B)$

Therefore

$$\boxed{P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}}$$