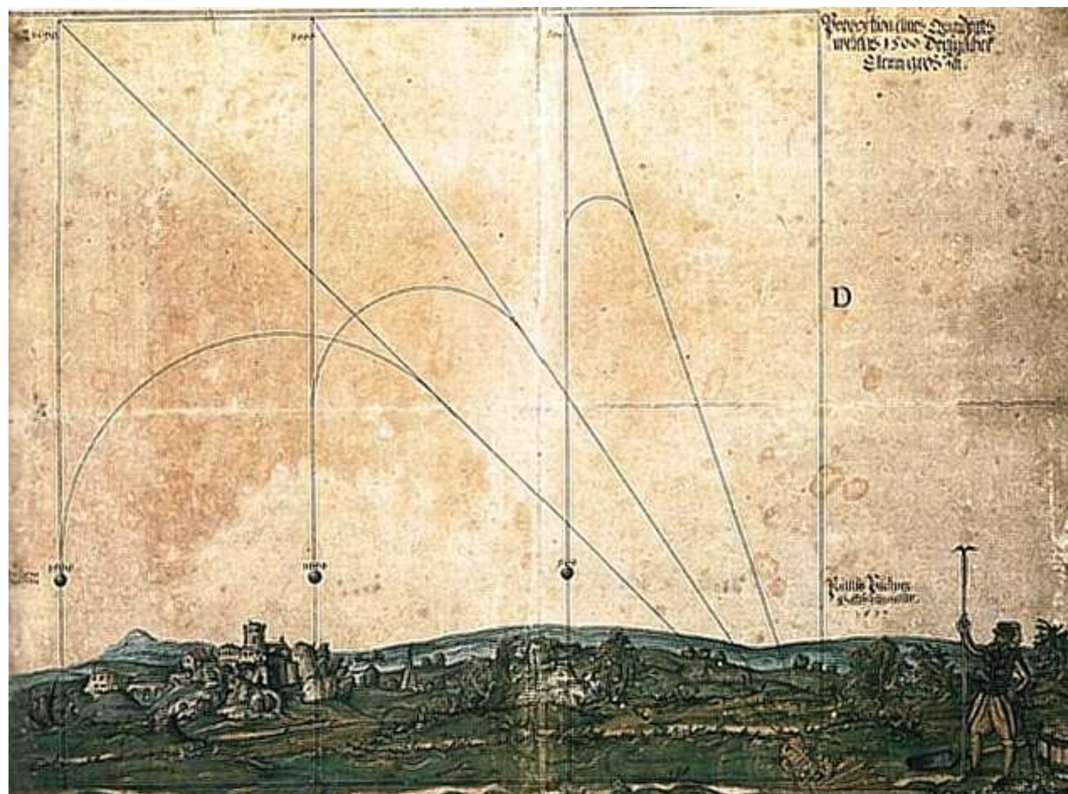
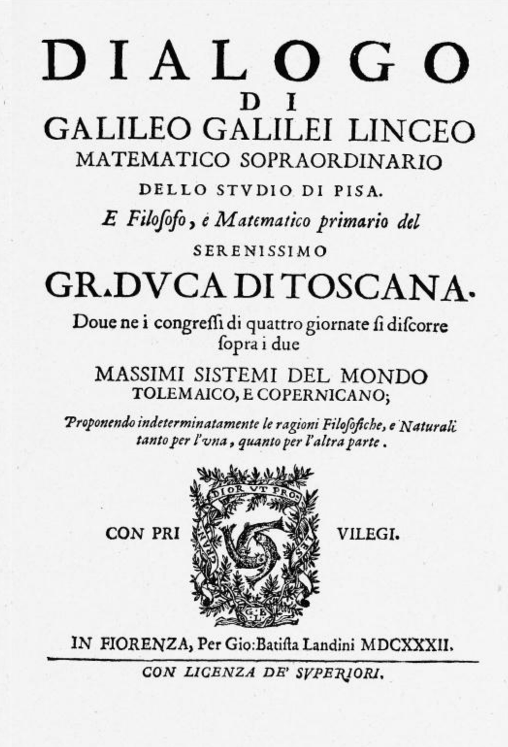
Lecture 4: Gravity

Aristotle:
Motion occurs in perfect geometrical figures – straight lines or circles.

Different objects fall at different speeds.
A heavy stone falls much faster than a light feather.





DIALOGO
DI
GALILEO GALILEI LINCEO
MATEMATICO SOPRAORDINARIO
DELLO STVDIO DI PISA.
E Filosofo, e Matematico primario del
SERENISSIMO
GR. DVCA DI TOSCANA.
Doue ne i congressi di quattro giornate si discorre
sopra i due
MASSIMI SISTEMI DEL MONDO
TOLEMAICO, E COPERNICANO;
*Proponendo indeterminatamente le ragioni Filosofiche, e Naturali
tanto per l'vna, quanto per l'altra parte.*

CON PRI VILEGI.

IN FIORENZA, Per Gio: Batista Landini MDCXXXII.

CON LICENZA DE' SVPERIORI.

Galileo (the experimentalist):
-tested it using objects dropped from a tower or rolled down an inclined plane

No, Aristotle, it does not.

1. *In a medium totally devoid of resistance all bodies will fall at the same speed.*
2. *During equal intervals of time a falling body receives equal increments of speed (i.e. it is accelerated by the same amount).*

The distance fallen is proportional to the square of the time

Newton's Laws of Motion

I. Every body maintains its state of rest or uniform motion in a straight line unless acted upon by an outside force.

II. The rate of change of momentum is proportional to the applied force

III. For every action there is an equal and opposite reaction

Mathematical expression of Newton's I & II Laws

$$F = \frac{d}{dt}(mv)$$

Mass (to be continued in lecture 5)

WHAT IS MASS ?

"Quantity of matter"

Force is defined in terms of $F = m\, a$

The unit of force is the *Newton*, defined as 1 kg m s^{-}

Inertial mass is a measure of an object's resistance to changing its state of motion when a force is applied.

The standard of mass is the value of a certain lump of matter held in a certain laboratory in Paris, denoted 1kg.

Originally the mass of I litre of distilled water.

How would you measure mass (without gravity) ?

Apply a known force and measure the acceleration . (use $F = ma$)

Consequences of Newton's Laws

Constant mass

$$F = m\frac{d}{dt}v = ma$$

If mass is constant

Newton's approach is therefore to FIND THE FORCES

If you know the force at any point during a particle's trajectory, you can calculate that trajectory

Gravitation as a universal force

- What types of forces are there?

- Newton wanted to unify Galileo's “natural” and “forced” motion - even if it meant introducing an “invisible” force that acts at a distance

- Newton's great insight - gravity is *universal* and applies not only to objects near the surface of the Earth but on all objects throughout space.

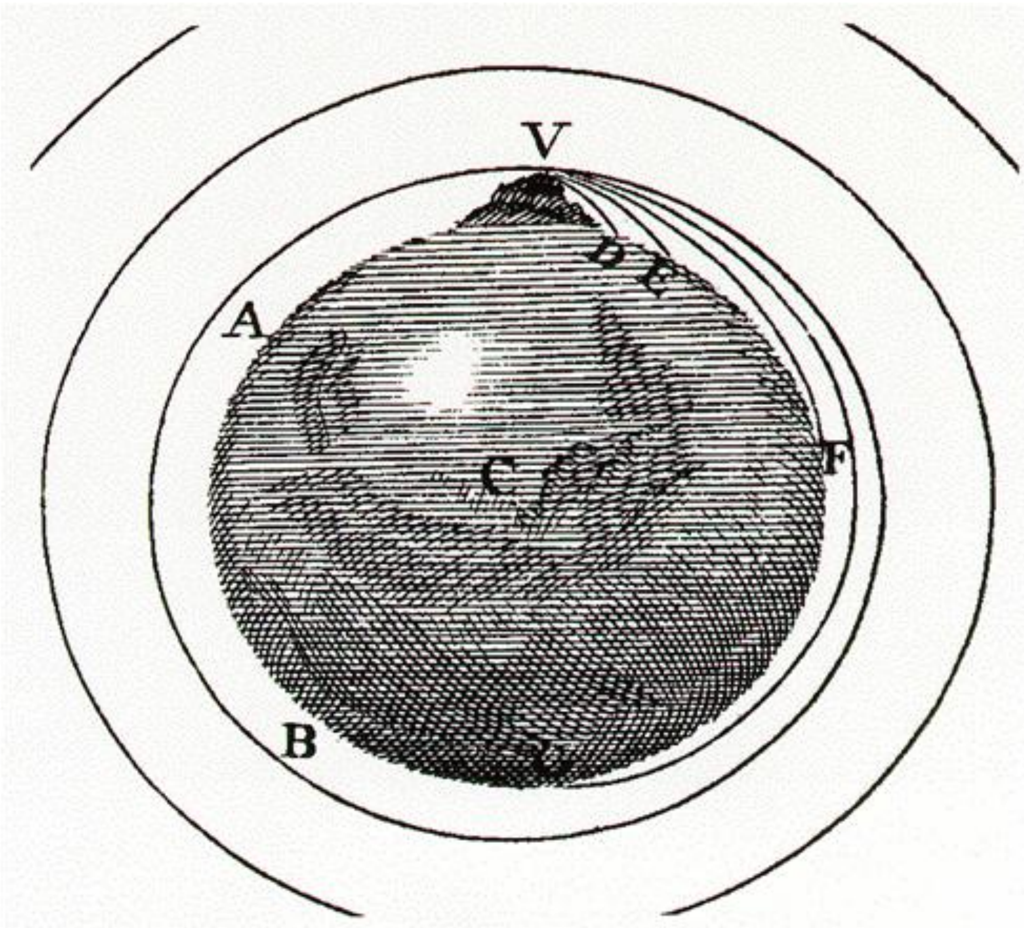
Newton called it gravitation *(gravitas)*

Assumption: *all* objects fall because there is a force (“gravity”) which attracts them to the centre of the Earth.

i.e. the centre of mass of the object is attracted to the centre of mass of the Earth

What happens if an object is thrown horizontally at greater and greater speeds?

Apples and heavenly bodies



Prediction: Heavenly bodies (such as the Moon) follow an orbit because they are continually "falling" towards the centre but "missing" the ground because of their tangential velocity.

What form will such a force have?

$F =$????

Galileo showed that gravity produces an acceleration that is independent of the mass of the object. From $F = m\,a$, this means that F must be proportional to the mass of the object.

Newton's Third Law states that if the Earth attracts an object, then the object attracts the Earth equally. Hence the mass of the Earth must also come into the equation.

Finally, the distance between the objects must be involved in some way. One would expect such a force to become weaker as the distance between the objects is increased.

If gravitation is truly universal, it holds as much for an apple falling to the surface of the Earth as it does for the moon "falling" round the Earth.

Period of rotation of moon = 27.3 days = 2358720 seconds

Angular velocity of moon ($2\pi/T$) = 0.00000027 rad s^{-1}

Distance of Moon from Earth = 384 000 km

Centripetal acceleration on moon ($\omega^2 r$) = 0.0027 m s^{-2}

Newton's Law of Gravity

Acceleration at Earth's surface = 9.8 m s^{-2}

Ratio of acceleration at surface and on moon = 3600 (actually 3629)

Distance of surface of Earth from centre = 6370 km

Ratio of distance of moon and Earth's surface = 60 (actually 60.3)

This ratio *squared* = 3600 (actually 3636)

Therefore:

$$F = G\frac{m_1 m_2}{r^2}$$

Applying it to the Solar System

If it works for the Moon, it should also work for the planets in the Solar system.

For a planet in a circular orbit: $F = m_p a = G\dfrac{m_s m_p}{r_p^2}$

so: $ar_p^2 = Gm_s = \text{constant}$

and $a = \omega^2 r_p = \left(\dfrac{2\pi}{T}\right)^2 r_p$

- Some results

Planet	Orbital Period x 10^7 s	Orbital Radius x 10^{11} m	Orbital Accel. x 10^{-3} ms^{-2}	$a\, r_p^{\,2}$ x 10^{20} $m^3 s^{-3}$
Mercury	0.760	0.579	39.6	1.33
Venus	1.94	1.08	11.3	1.32
Earth	3.16	1.50	5.93	1.33
Mars	5.94	2.28	2.55	1.33
Jupiter	37.4	7.78	0.219	1.33
Saturn	93.5	14.3	0.0646	1.32
Uranus	264	28.7	0.0163	1.34
Neptune	522	45.0	0.00652	1.32
Pluto	782	59.1	0.00382	1.33

Taking $m_s = 1.99 \times 10^{30}$ kg, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

The quest for unification of forces

Newton *unified* terrestrial and celestial **gravitation**.

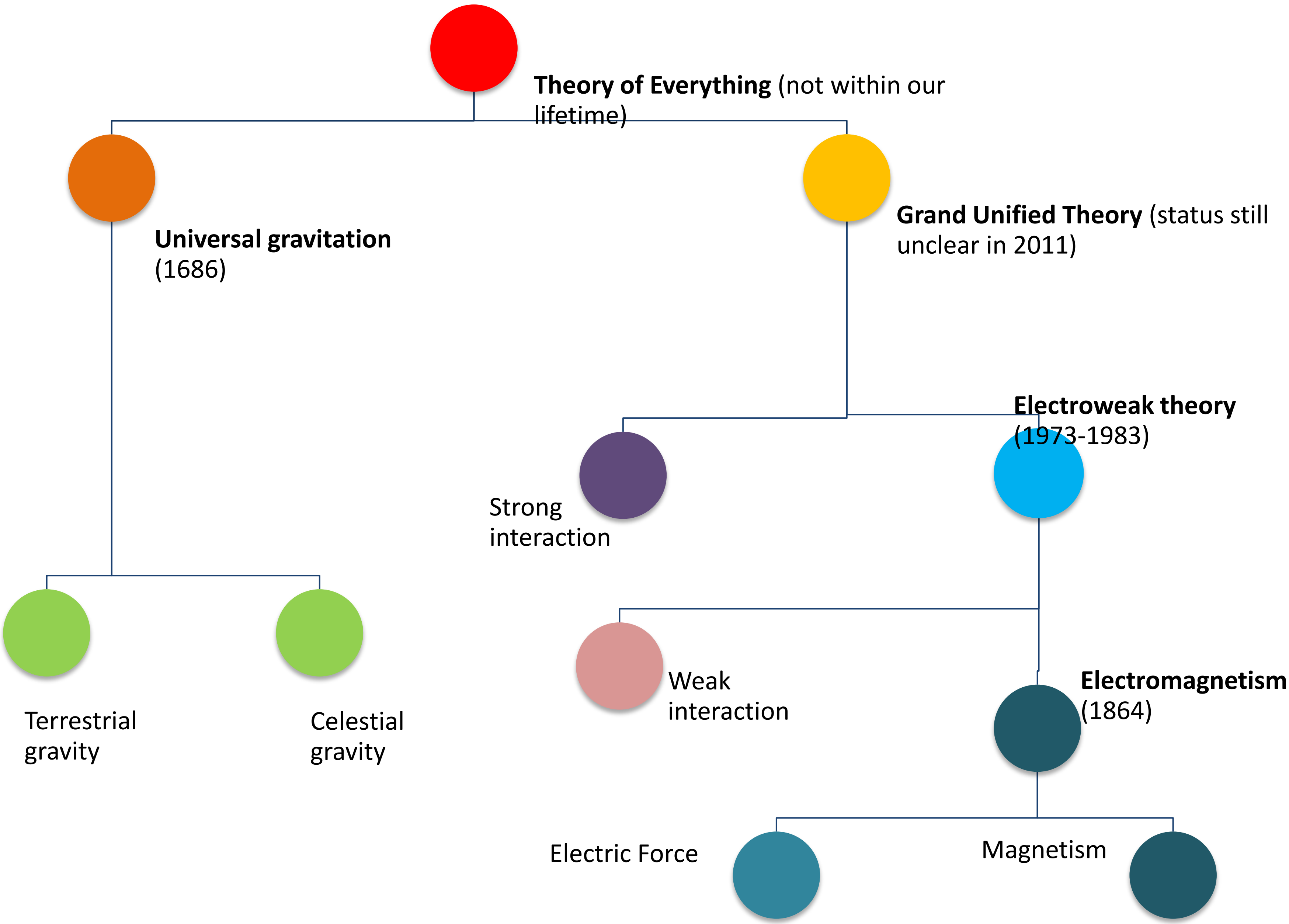
The discovery of electromagnetism and advances in chemistry and materials science brought the realisation that the majority of forces we are in contact with are **electromagnetic** in nature.

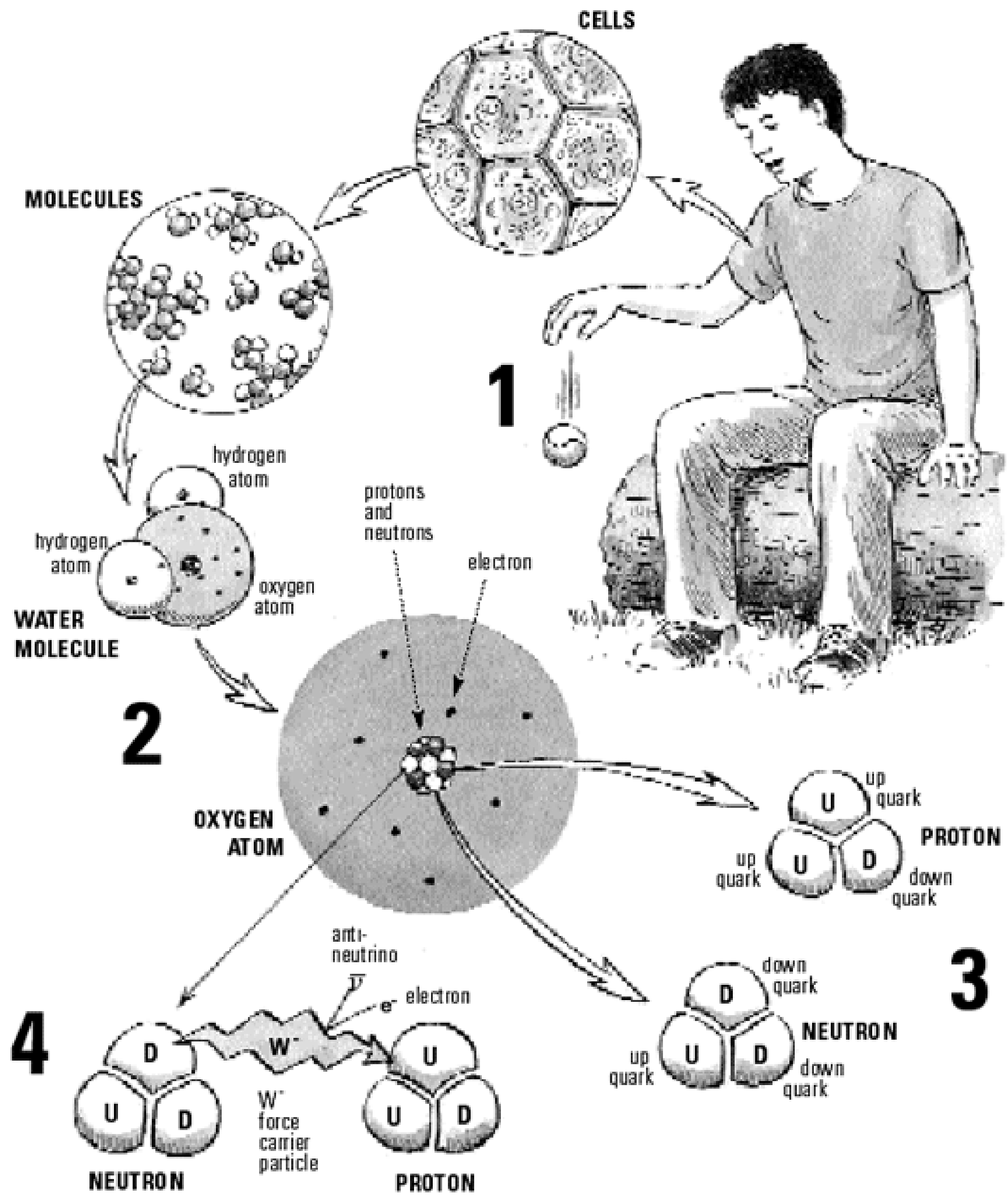
Later advances in nuclear physics required the existence of two nuclear forces: the **strong force** that protons and neutrons within the nucleus, and the **weak force** that mediates the decay of certain particles.

These four forces are sufficient to explain *all* known physical phenomena:

The fundamental forces *all* involve action-at-a-distance

A scheme for unification




CELLS
MOLECULES
1
hydrogen atom
hydrogen atom
oxygen atom
WATER MOLECULE
protons and neutrons
electron
2
OXYGEN ATOM
U
up quark
PROTON
up quark
U
D
down quark
3
D
down quark
NEUTRON
up quark
U
D
down quark
4
anti-neutrino
$\bar{\nu}$
e^- electron
D
W^-
U
U
D
W^- force carrier particle
U
D
NEUTRON
PROTON