Lecture 6: The Equivalence Principle

- What is the Equivalence Principle?
- Why do we want to test it?
- How has it been tested?
- Results

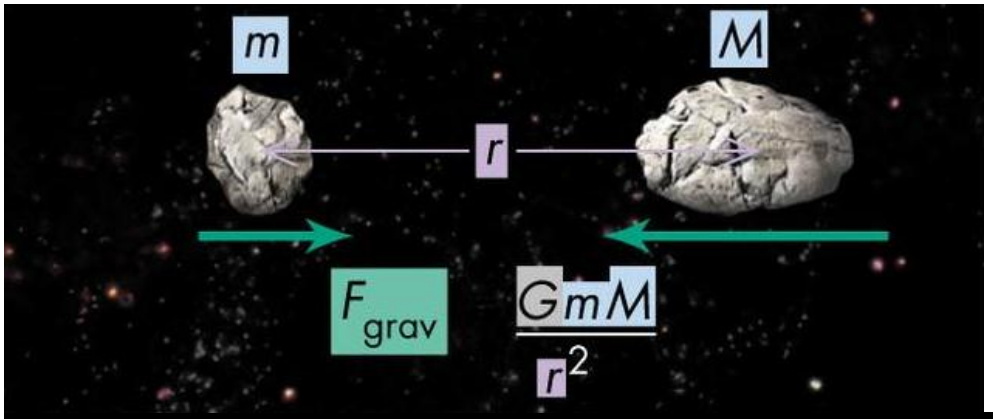
Both **Newtonian physics** and **General Relativity** are based on the **Equivalence Principle:**

All bodies fall in a gravitational field with the same acceleration regardless of their mass or internal structure.

- Later, Einstein realized that a uniform gravitational field is the same as an accelerated reference frame. More on this in later lectures.

Two different definitions for mass

- Recall the concept of momentum and of potentials and fields
- The **inertial mass** is the resistance of an object to a change in momentum.
- **Gravitational mass** is a property of the mass of an object that produces a gravitational field in the space surrounding the object.



Most important formula for physics and mathematics

$$m_{\mathrm{i}} \ddot{\boldsymbol{x}} = \boldsymbol{F}$$

Model for force

$$\boldsymbol{F} = -m_{\mathrm{g}} \boldsymbol{\nabla} U \qquad U = \text{Newton potential}$$

Acceleration

$$\boldsymbol{a} = \ddot{\boldsymbol{x}} = -\frac{m_{\mathrm{g}}}{m_{\mathrm{i}}} \boldsymbol{\nabla} U$$

Experiment

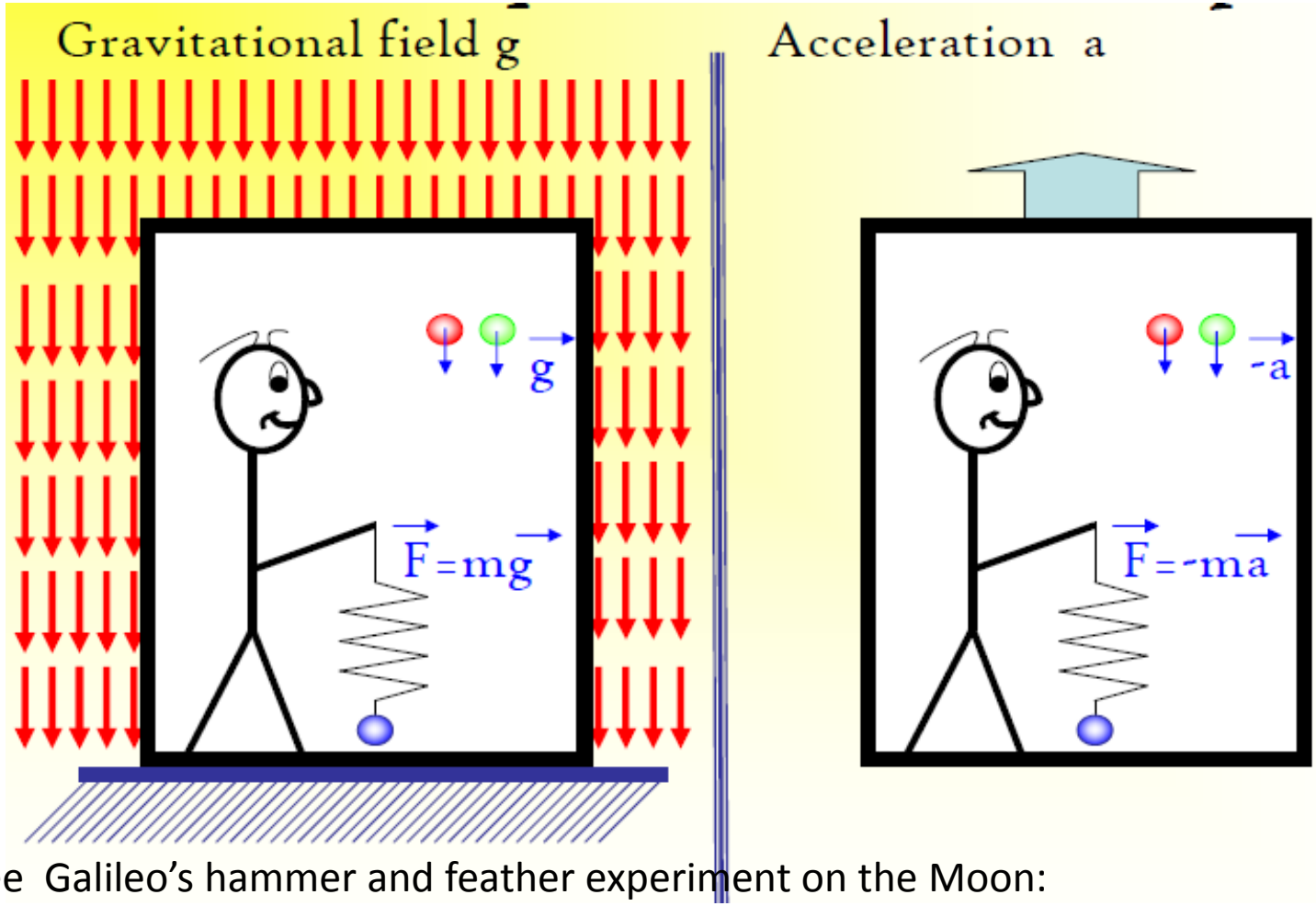
$$\eta = \frac{a_2 - a_1}{\frac{1}{2}(a_1 + a_1)} = \frac{(m_{\mathrm{g}}/m_{\mathrm{i}})_2 - (m_{\mathrm{g}}/m_{\mathrm{i}})_1}{\frac{1}{2}\left((m_{\mathrm{g}}/m_{\mathrm{i}})_2 + (m_{\mathrm{g}}/m_{\mathrm{i}})_1\right)} \leq 2 \cdot 10^{-13}$$

Idealization: Equivalence Principle $m_{\mathrm{g}} = m_{\mathrm{i}}$

The Equivalence Principle:

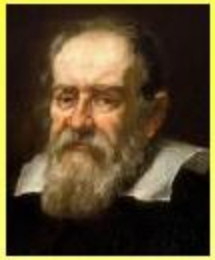
inertial mass = gravitational mass

$m_I = m_G$ for all bodies



See Galileo's hammer and feather experiment on the Moon:
http://nssdc.gsfc.nasa.gov/planetary/image/featherdrop_sound.mov

Historical overview



Bessel

$$\eta = \frac{|a_1 - a_2|}{\frac{1}{2}(a_1 + a_2)}$$

Eötvös parameter

Galileo

year

1600 1700 1800 1900 2000

Type of experiment

drop

pendula

torsion balance

modulated torsion balance

10^{-3}

10^{-6}

10^{-9}

10^{-12}

η

Potter

Newton

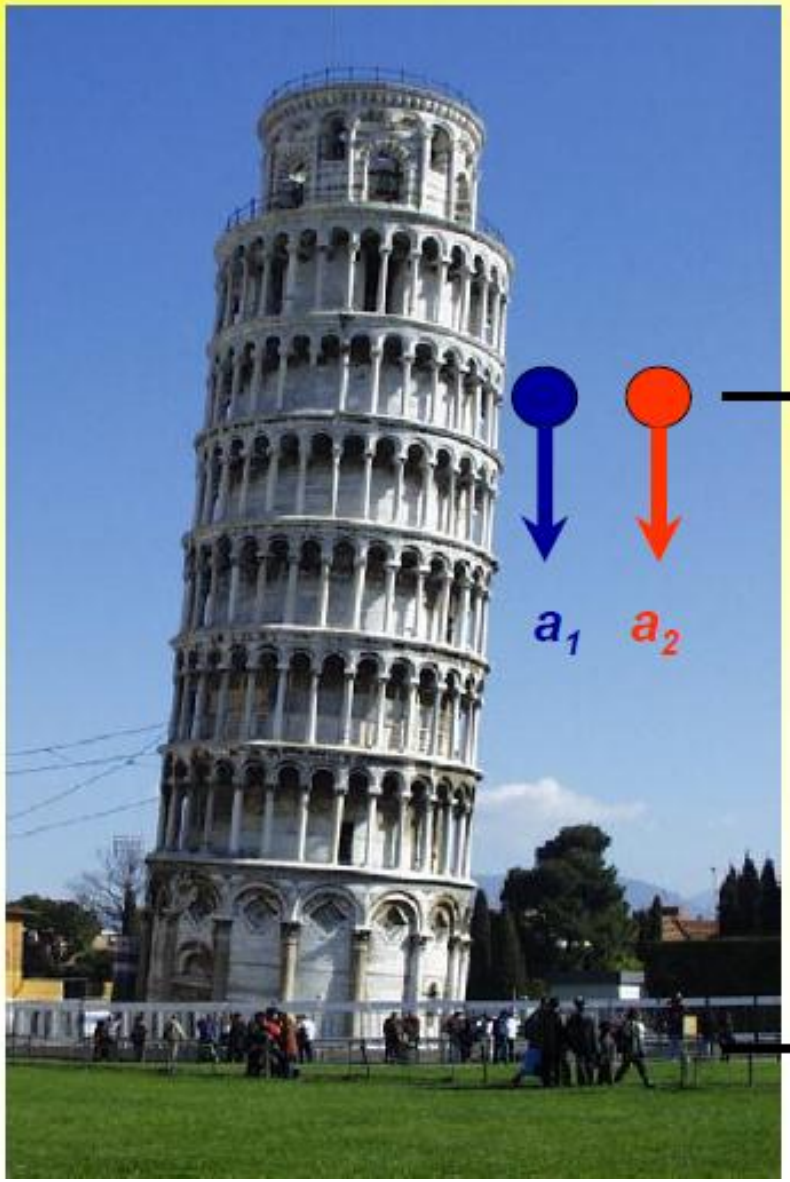
Eötvös

Dicke

UW

Braginsky

First tests of the Equivalence Principle



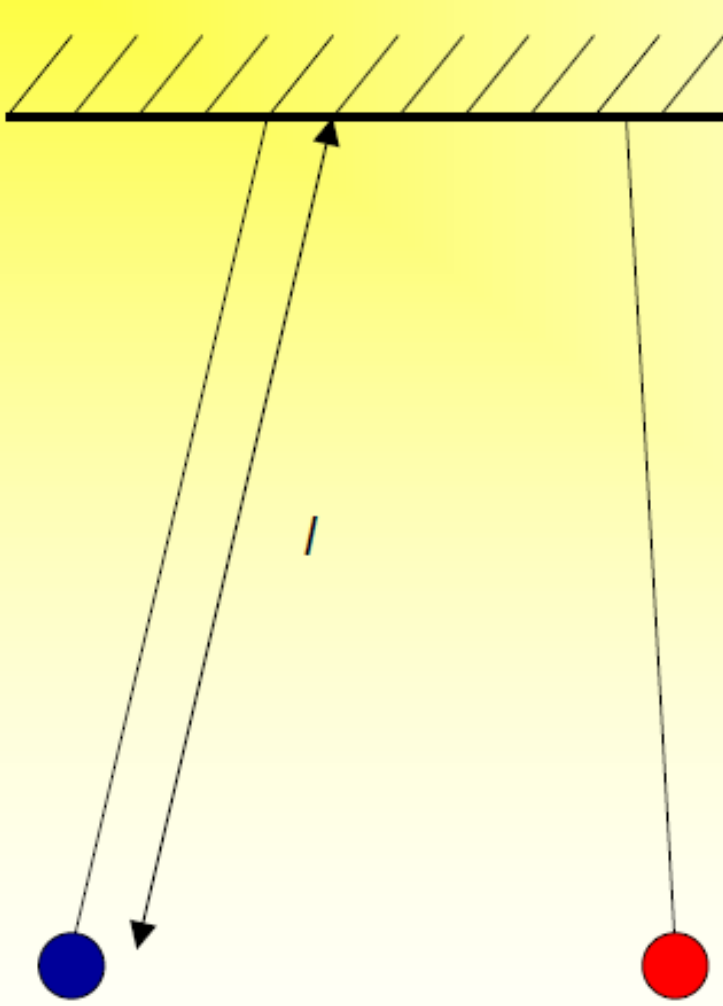
$$F = m_G g$$
$$F = m_I a$$
$$a = \frac{m_G}{m_I} g$$

Time t to fall from h:

$$t = \sqrt{\frac{2h}{m_G / m_I \, g}}$$

1600 Galileo: $\eta = \frac{a_1 - a_2}{\frac{1}{2}(a_1 + a_2)} \approx 0.1$

Second generation tests



Measurement of the swing periods of pendula:

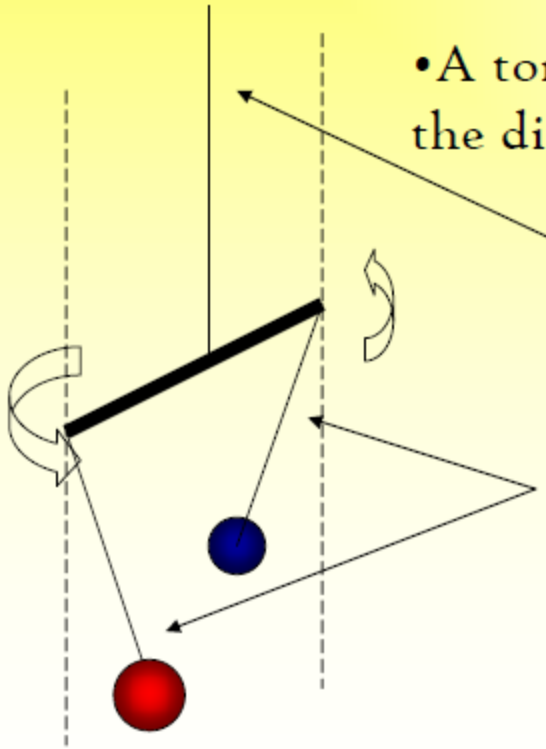
$$T = 2\pi \sqrt{\frac{L}{g}\frac{m_I}{m_G}}$$

Newton (1686), Bessel (183), Porter (1923)

$$\eta \approx 2x10^{-5}$$

Third generation: Eötvös experiments

- A violation of the EP would yield to different plumb-line for different materials.
- A torsion balance can be used to measure the difference in plumb-lines:

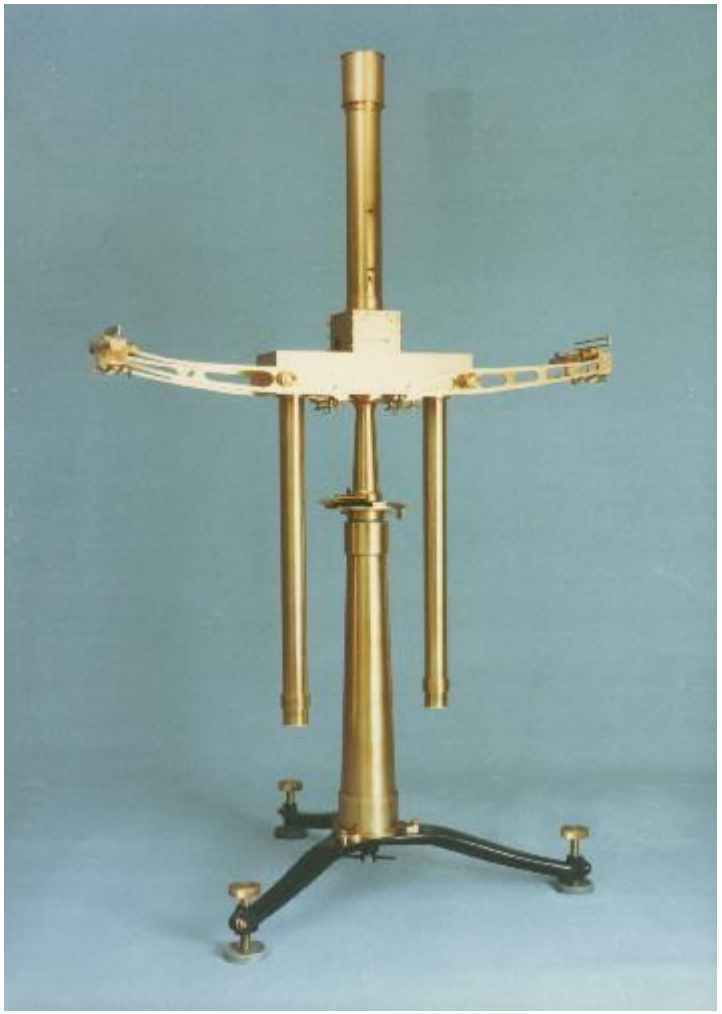


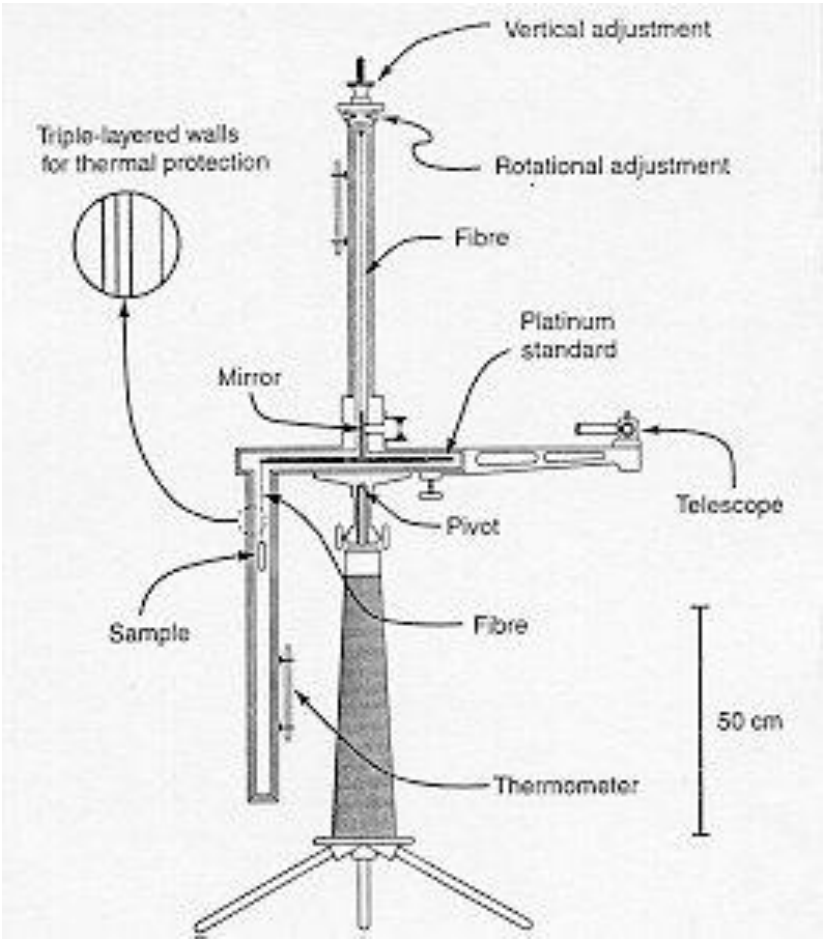
Torsion fiber hangs like the average plumb line.

Difference in plumb lines produces a torque on the beam.

Eötvös (1922) $\eta \approx 5x10^{-9}$

Torsion balance: Loránd Eötvös (1885)





Important results

Universality principles

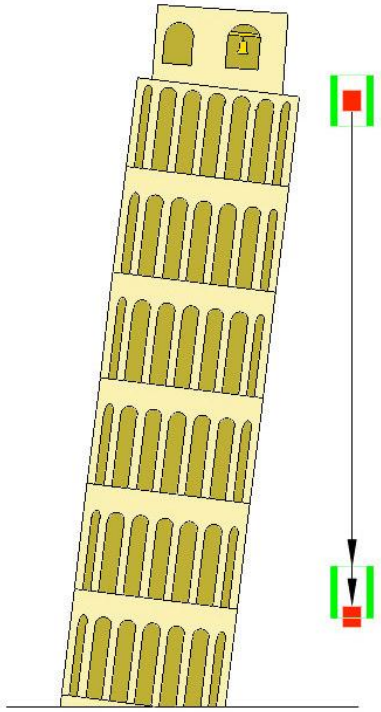
- Gravity acts on all kinds of matter
- Gravity acts on all kinds of matter in the same way
- Gravity acts on all kinds of clocks
- Gravity acts on all kinds of clocks in the same way
- Gravity is created from all kinds of matter
- Gravity is created from all kinds of matter in the same way

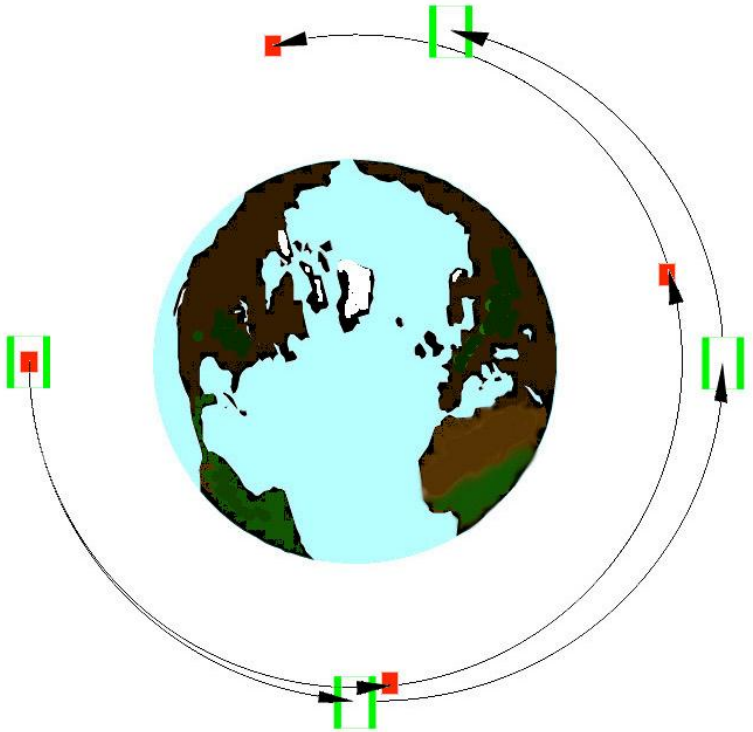
Table V. EQUIVALENCE PRINCIPLE TESTS

Year	Investigator	Accuracy	Method
500?	Philoponus[20]	"small"	Drop Tower
1585	Stevin[19]	$5 \cdot 10^{-2}$	Drop Tower
1590?	Galileo[2]	$2 \cdot 10^{-2}$	Pendulum, Drop Tower
1686	Newton[3]	10^{-3}	Pendulum
1832	Bessel[21]	$2 \cdot 10^{-5}$	Pendulum
1910	Southerns[22]	$5 \cdot 10^{-6}$	Pendulum
1918	Zeeman[23]	$3 \cdot 10^{-8}$	Torsion Balance
1922	Eötvös[24]	$5 \cdot 10^{-9}$	Torsion Balance
1923	Potter[25]	$3 \cdot 10^{-6}$	Pendulum
1935	Renner[26]	$2 \cdot 10^{-9}$	Torsion Balance
1964	Dicke,Roll,Krotkov[27]	$3 \cdot 10^{-11}$	Torsion Balance
1972	Braginsky,Panov[28]	10^{-12}	Torsion Balance
1976	Shapiro, et al.[29]	10^{-12}	Lunar Laser Ranging
1981	Keiser,Faller[30]	$4 \cdot 10^{-11}$	Fluid Support
1987	Niebauer, et al.[31]	10^{-10}	Drop Tower
1989	Heckel, et al.[32]	10^{-11}	Torsion Balance
1990	Adelberger, et al.[33]	10^{-12}	Torsion Balance
1999	Baeßler, et al.[34]	$5 \cdot 10^{-13}$	Torsion Balance
2010?	MiniSTEP[35]	10^{-17}	Earth Orbit

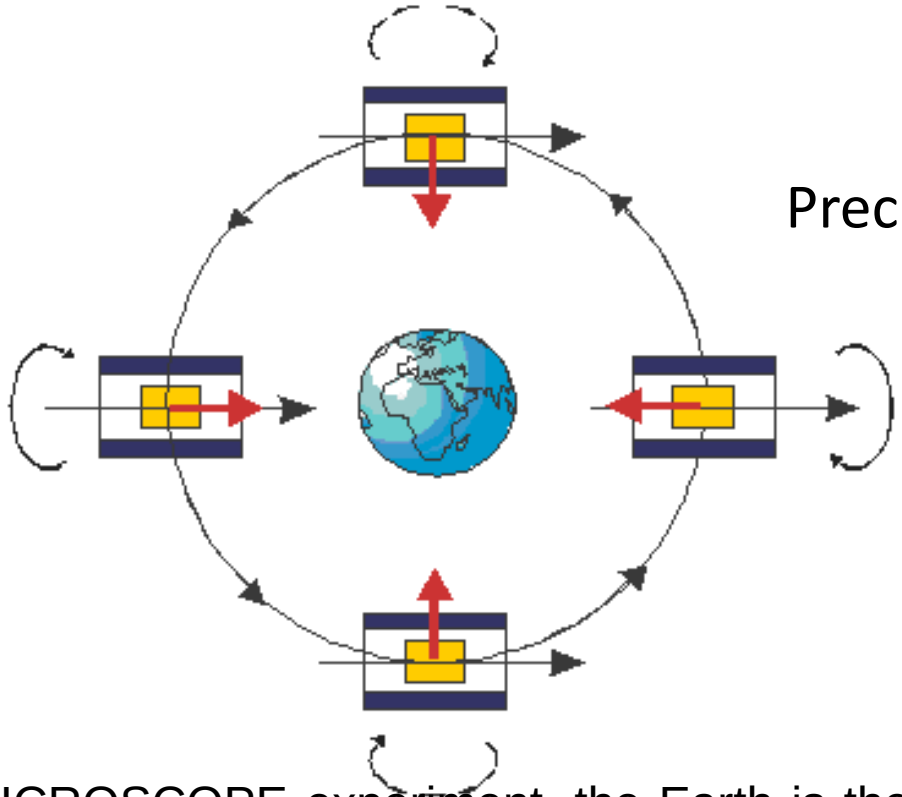
Ciufolini & Wheeler *Gravitation and Inertia* (Princeton University Press: Princeton, 1995) pp. 117-119

Fourth generation: Torsion balance in space (no strings attached)





MICROSCOPE: A future high-precision experiment



Precision: Better than 10^{-18}

In the MICROSCOPE experiment, the Earth is the gravitational source about which free fall motion of two masses, composed of different materials, is observed and controlled taking care that both masses are submitted exactly to the same gravitational field. The controlled electrostatic field, added to break the experimentation symmetry by forcing the masses to remain on the same orbit is accurately measured: a defect of symmetry gives rise to evidence of an EP violation.

Why test the Equivalence Principle?

The Equivalence Principle is intimately connected with some of the fundamental aspects of modern physics, and of the unification of gravity with particle physics.

- It is a test of fundamental constants of physics.
- It is a test of the **coupling constant** between mass and the gravitational field.
- EP experiments are testing a key assumption of some new physical paradigms that are being proposed, such as string theory.

Some inspiration from Eötvös

Laurels beckoned us, so we started out
With Nightingale towards a mountain height.
While I grappled with the sheer cliffs below,
She seized her prize in easy, graceful flight.

What I may perhaps never ever reach,
Took but a brief moment for the bird;
O Heaven don't be so unjust, I plead,
Grant me wings too. Let my prayer be heard.