Lecture 7: Frames of reference

The concept of reference frames is central to Relativity, as will be seen in later lectures. Keep it in mind.

- An **event** is something that occurs at a precisely determined point in space and time.

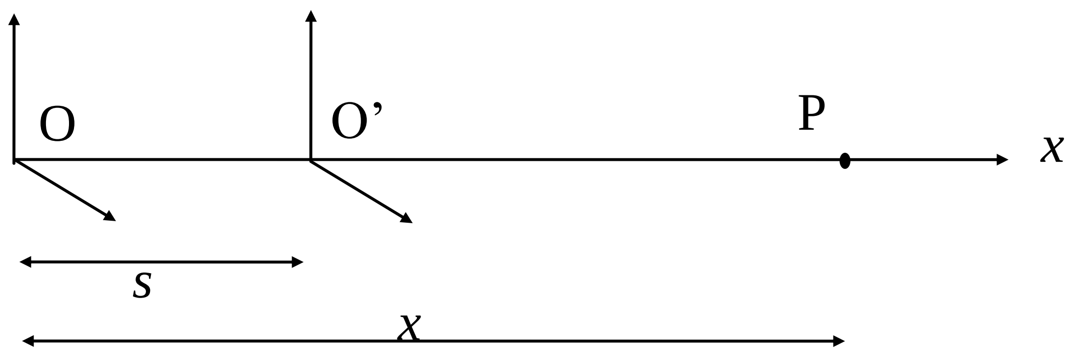
Its location is therefore determined by 4 coordinates (x,y,z,t) in a 4-dimensional **space-time.**

Question: How do different observers (in different *reference frames*) locate the event? i.e. **what coordinates do they assign to the event?**

This question is really a continuation of our earlier question on how we measure position and time. If we use rulers and clocks, how do we calibrate our measuring instruments?

Case 1: Stationary observers displaced in space

O and O’ are two observers in space-time.



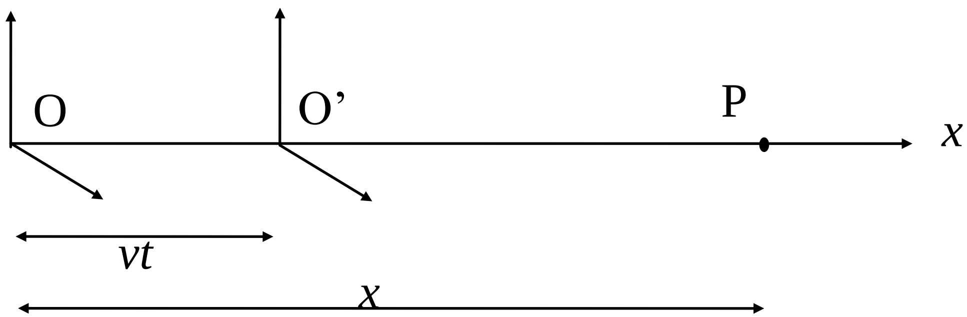
$$x' = x - s \quad \text{(Geometry)}$$
$$y' = y \quad \text{(Geometry)}$$
$$z' = z \quad \text{(Geometry)}$$
$$t' = t \quad \textbf{(Assumption)}$$

The assumption of **universal time** defines what is known as **Galilean relativity**.

Later on we will see how this assumption is dropped in Special Relativity.

Case 2 - Moving observers (constant velocity)

Suppose O' is moving with constant velocity *v* along the x-axis and O is stationary. Suppose also that O and O' coincide at t=0



$x' = x - vt$	(Geometry)
$y' = y$	(Geometry)
$z' = z$	(Geometry)
$t' = t$	**(Assumption)**

Note: There is no real reason to suppose O is stationary and O' is moving: it could be the other way round. The correct statement is to say that **O' has a velocity *v* relative to O.**

Case 3: Velocity measured by moving observers (at constant velocity)

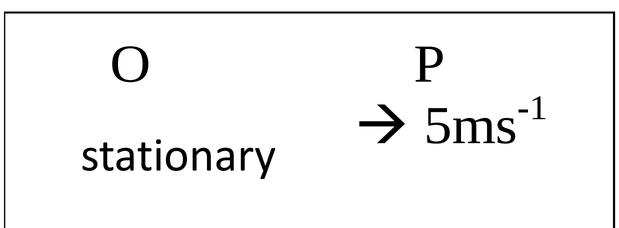
Using differentiation:

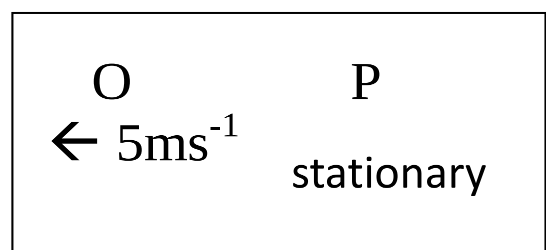
$$\frac{d}{dt}x' = \frac{d}{dt}x - \frac{d}{dt}vt$$

therefore:

$$v_x' = v_x - v$$
$$v_y' = v_y$$
$$v_z' = v_z$$
$$t' = t$$

- The velocity of an object seen by an observer moving in the same direction is the **difference** of their velocities.
- This is the rule governing relative velocities in classical mechanics.





Case 4: Acceleration measured by moving observers (at constant velocity)

Differentiating again:

$$\frac{d}{dt}v_x' = \frac{d}{dt}v_x - \frac{d}{dt}v$$

$a_x' = a_x$
$a_y' = a_y$
$a_z' = a_z$
$t' = t$

- In other words, observers moving at constant velocity relative to one another ***(inertial*** **observers) will agree on accelerations, and hence, forces.**
- **This is the principle of classical (Galilean) relativity.**
- The equations for *x, v* and *a* are called the **Galilean transformations.**

Summary: Newtonian Relativity

1. There exists an **absolute space** in which Newton's laws are true. An inertial frame is a reference frame in relative uniform motion to absolute space.
2. All inertial frames share a **universal time**.

Galilean relativity can be shown as follows. Consider two inertial frames S and S' . A physical event in S will have position coordinates $r = (x, y, z)$ and time t; similarly for S' . By the second axiom above, one can synchronize the clock in the two frames and assume $t = t'$. Suppose S' is in relative uniform motion to S with velocity v. Consider a point object whose position is given by $r = r(t)$ in S. We see that

$$r'(t) = r(t) - vt.$$

The velocity of the particle is given by the time derivative of the position:

$$u'(t) = \frac{d}{dt}r'(t) = \frac{d}{dt}r(t) - v = u(t) - v.$$

Another differentiation gives the acceleration in the two frames:

$$a'(t) = \frac{d}{dt}u'(t) = \frac{d}{dt}u(t) - 0 = a(t).$$

These are the **Galilean transformations.**

- The laws of motion are the same when measured in all inertial frames.
- There is no way of telling whether an inertial frame is moving or stationary.
- There is no “stationary” frame.

Non-inertial frames of reference

What happens when O' is accelerating?

Taking the x-coordinate only:

$$x' = x - s$$

$$= x - u\,t - \tfrac{1}{2}\,a\,t^2$$

Differentiating,

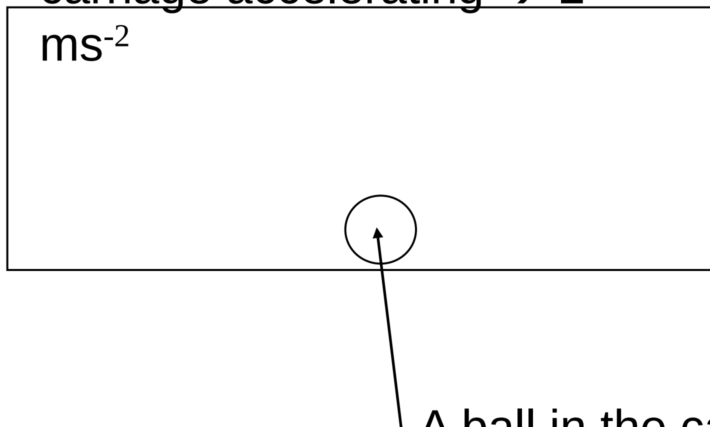
$$v_x' = v_x - u - at$$

$$a_x' = a_x - a$$

So the acceleration of a point is seen by an observer accelerating in the same direction as the **difference** of the two accelerations.

But a different acceleration implies a different force
=> **the laws of motion are not the same in accelerating (*non-inertial*) frames of reference**

<u>Example</u>



A ball in the carriage is:

- stationary as seen by a stationary observer outside the carriage
- accelerating ← 2 ms^{-2} as seen by an observer inside the carriage

However

<u>Can</u> the observer inside the carriage say that it is moving (without looking outside)?

How can the observer inside the carriage explain his observations if he does not know the carriage is moving?

=> He will deduce that there is a force acting on the ball accelerating it at ← 2 ms^{-2}

A hypothetical *pseudo-force* can make Newton's laws of motion hold in a non-inertial frame. The force ***F*** does not arise from any physical interaction but rather from the acceleration ***a*** of the non-inertial reference frame itself.

Questions to think over (and research topics in brackets):

- Why do you feel heavier when a lift starts moving upwards?

- Why will you be "weightless" if you are in a plane accelerating downwards at 9.8 ms^{-2} ?

- Is a rotating frame of reference inertial? (Mach's Principle)

- Does centrifugal force exist?

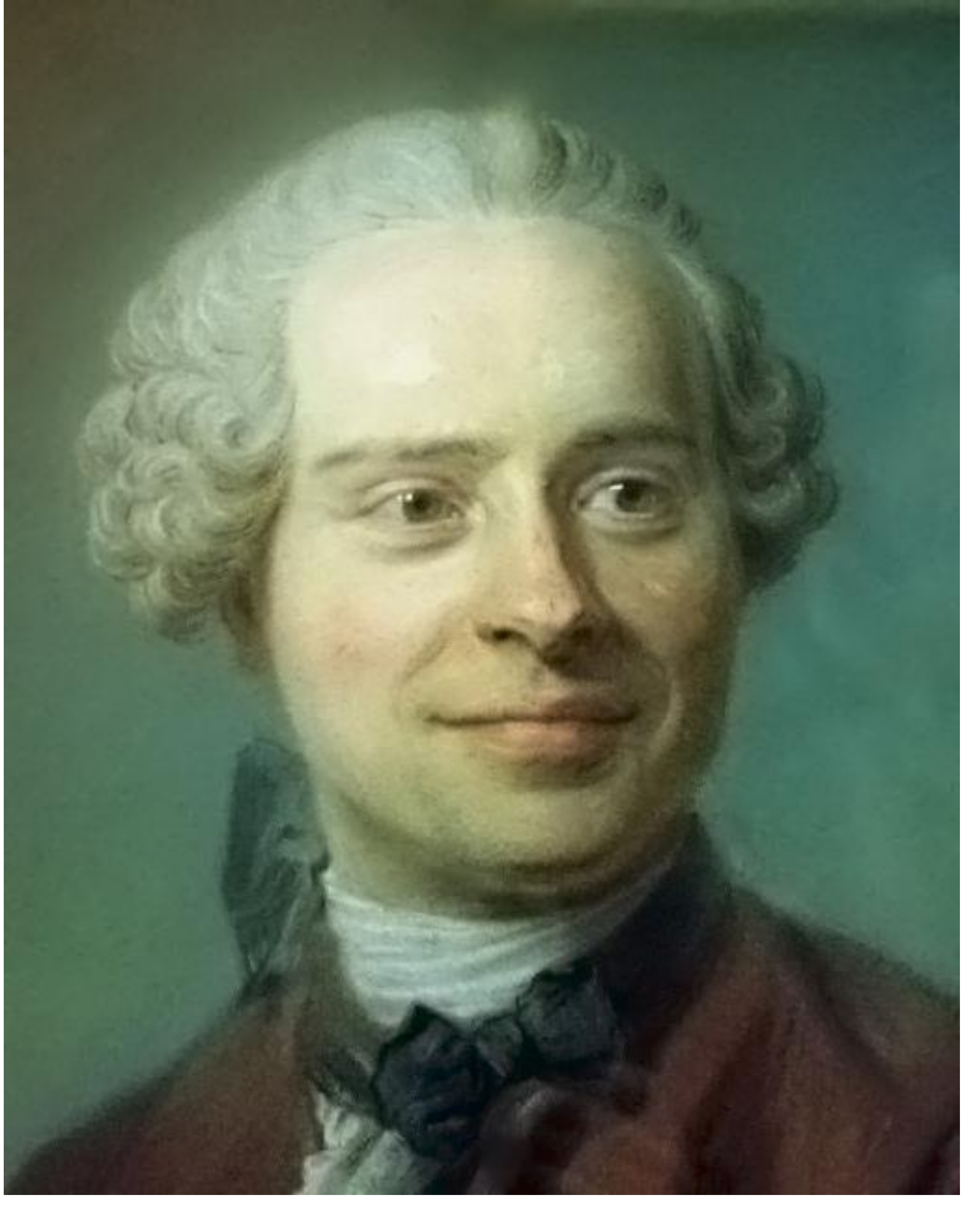
- What are the pseudo-forces in a rotating frame of reference? (Coriolis force)

- Why are astronauts in orbit "weightless"? (Is "zero-g" a correct description for this state?)

- Is gravitation a pseudo-force? (General Relativity)

Physicist of the week

The notion of a pseudo-force was formulated by d’Alembert (d’Alembert’s Principle)



Jean-Baptiste le Rond d'Alembert
1717 – 1783