Lecture 8 part II: Special Relativity

The two postulates of Special Relativity:

1. The speed of light is the same for all observers, no matter what their relative speeds.

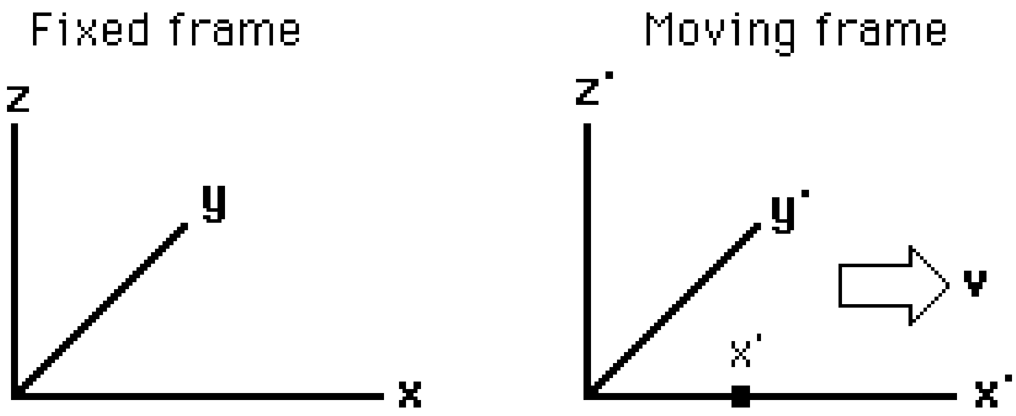
2. The laws of physics are the same in any inertial (that is, non-accelerated) frame of reference.

Basic difference between the two theories of physics:

- Newtonian mechanics: uses **Galilean transformation**

- Special Relativity: **Lorentz transformation**

Lorentz transformation



The primed frame moves with velocity v in the x direction with respect to the fixed reference frame. The reference frames coincide at t=t'=0. The point x' is moving with the primed frame.

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The reverse transformation is:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

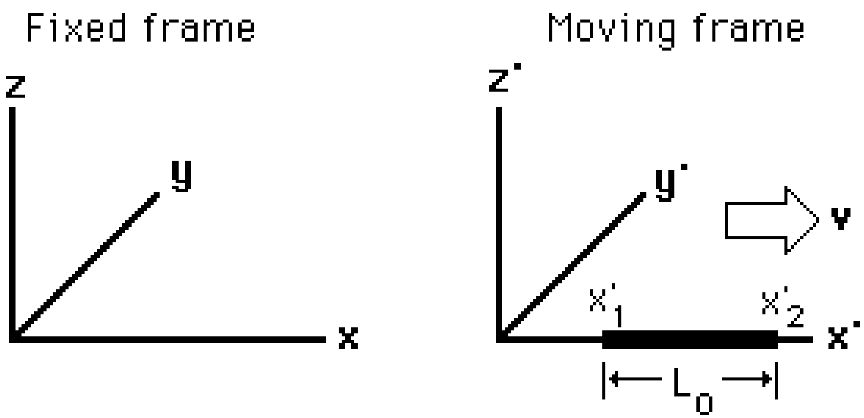
$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is the **Lorentz factor**

Length contraction



The length of any object in a moving frame will appear foreshortened in the direction of motion, or contracted. The amount of contraction can be calculated from the **Lorentz transformation**.
The length is maximum in the frame in which the object is at rest.

If the length $L_0 = x'_2 - x'_1$ is measured in the moving reference frame, then $L = x_2 - x_1$ can be calculated using the Lorentz transformation.

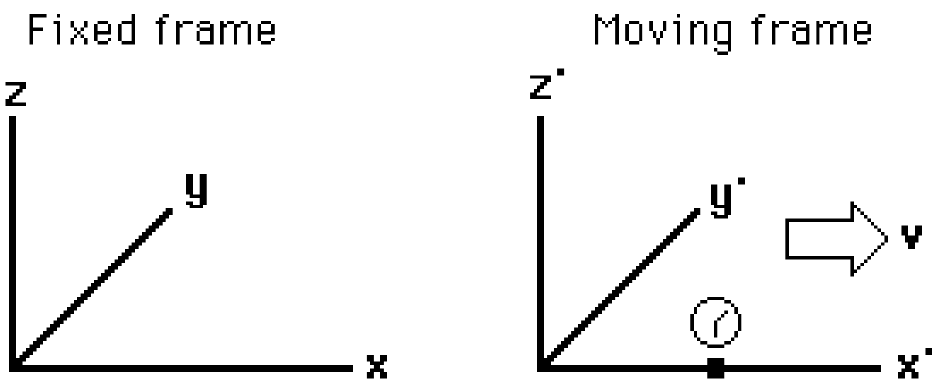
$$L_0 = x'_2 - x'_1 = \frac{x_2 - vt_2 - x_1 + vt_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

But since the two measurements made in the fixed frame are made simultaneously in that frame, $t_2 = t_1$, and

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$

Length contraction

Time dilation



A clock in a moving frame will be seen to be running slow, or "dilated" according to the Lorentz transformation.
The time will always be shortest as measured in its rest frame. The time measured in the frame in which the clock is at rest is called the **proper time**.

If the time interval $T_0 = t'_2 - t'_1$ is measured in the moving reference frame, then $T = t_2 - t_1$ can be calculated using the Lorentz transformation.

$$T = t_2 - t_1 = \frac{t'_2 + \frac{vx'_2}{c^2} - t'_1 - \frac{vx'_1}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The time measurements made in the moving frame are made at the same location, so the expression reduces to:

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} = T_0 \gamma$$

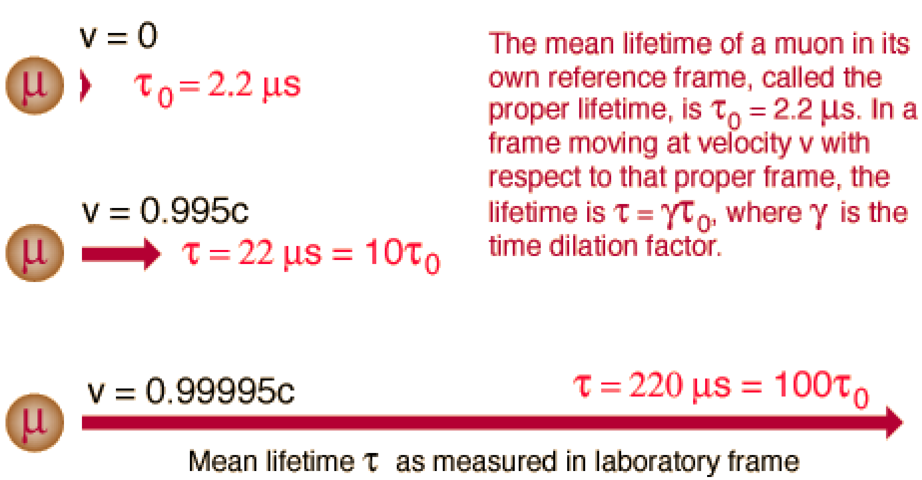
Time dilation experiments

Twin Paradox

The story is that one of a pair of twins leaves on a high speed space journey during which he travels at a large fraction of the speed of light while the other remains on the Earth. Because of time dilation, time is running more slowly in the spacecraft as seen by the earthbound twin and the traveling twin will find that the earthbound twin will be older upon return from the journey. **The common question: Is this real? Would one twin really be younger?**

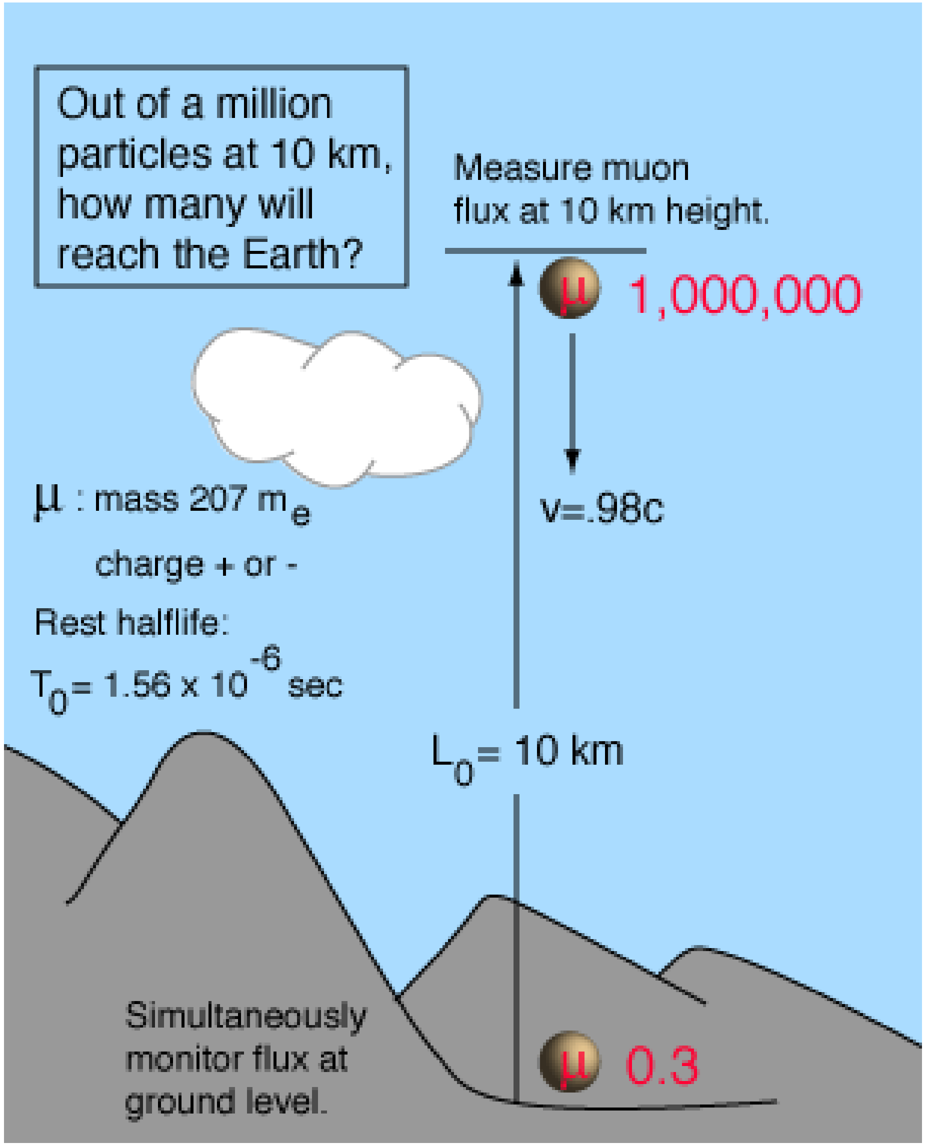
The basic question about whether time dilation is real is settled by the **muon experiment**. **The clear implication is that the traveling twin would indeed be younger**, but the scenario is complicated by the fact that the traveling twin must be accelerated up to traveling speed, turned around, and decelerated again upon return to Earth. Accelerations are outside the realm of Special Relativity and require **General Relativity**.

Despite the experimental difficulties, an experiment on a commercial airliner confirms the existence of a time difference between ground observers and a reference frame moving with respect to them.



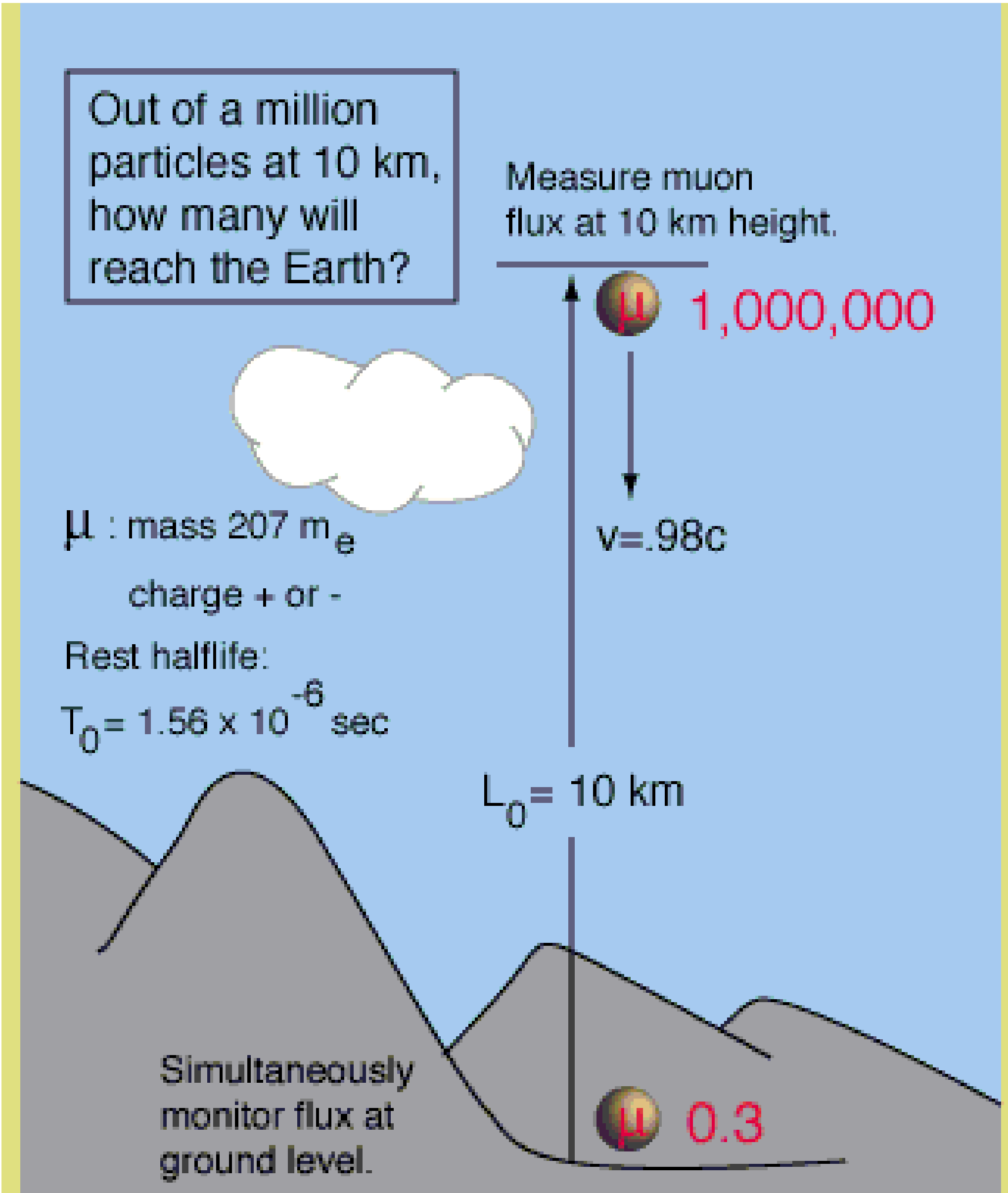
The mean lifetime of a muon in its own reference frame, called the proper lifetime, is $\tau_0 = 2.2\ \mu s$. In a frame moving at velocity v with respect to that proper frame, the lifetime is $\tau = \gamma\tau_0$, where γ is the time dilation factor.

Muon experiment: Experimental verification of time dilation and length contraction



The measurement of the flux of muons at the Earth's surface produced an early dilemma because many more are detected than would be expected, based on their short half-life of 1.56 microseconds.

Non-Relativistic



Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{10^4\ \text{m}}{(0.98)(3 \times 10^8\ \text{m/s})}$

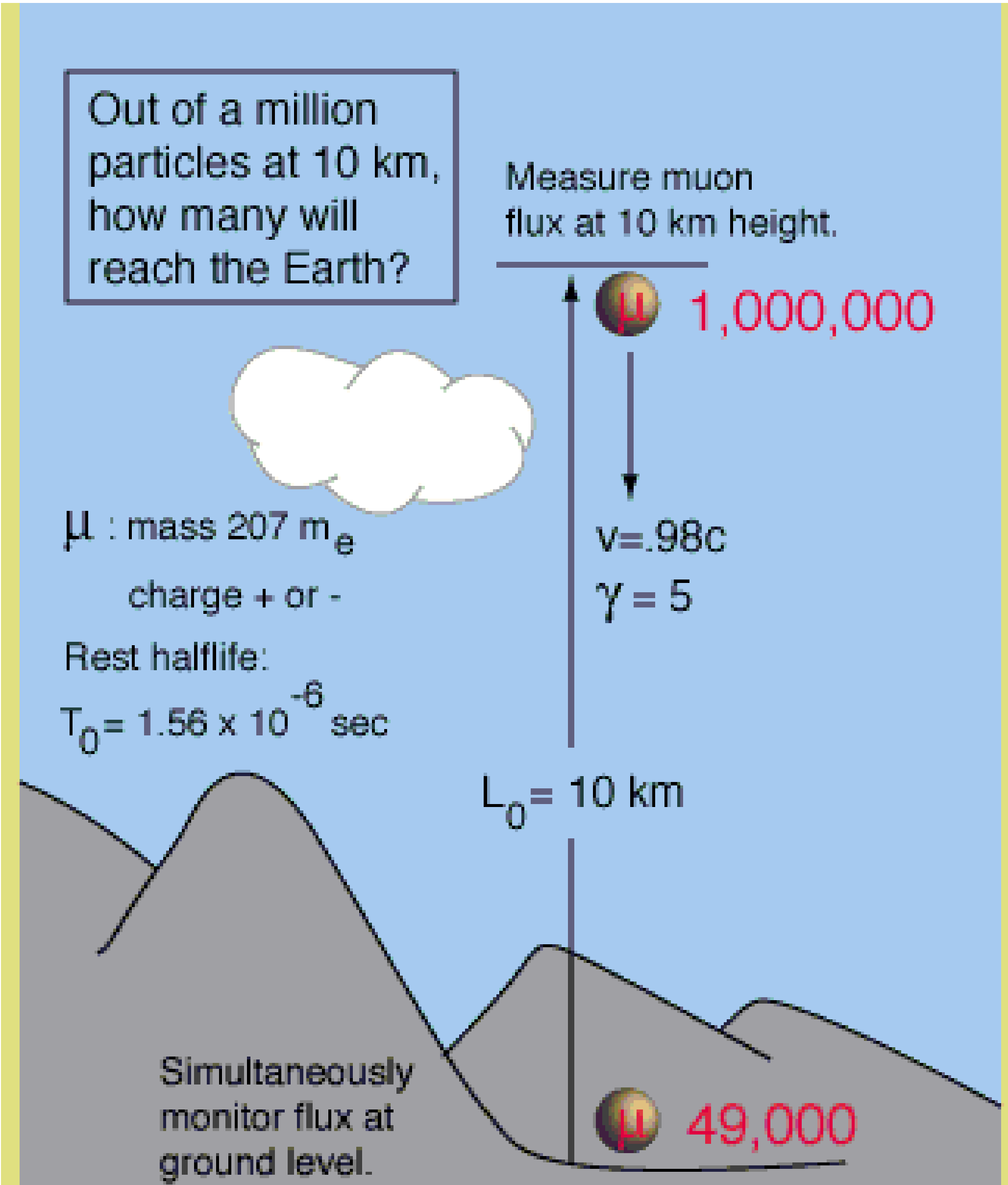
$T = 34 \times 10^{-6}\ \text{s} = 21.8$ halflives

Survival rate:

$$\frac{I}{I_0} = 2^{-21.8} = 0.27 \times 10^{-6}$$

Or only about 0.3 out of a million.

Relativistic, Earth-Frame Observer



Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{10^4 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

$T = \; 34 \times 10^{-6}$ s = 4.36 halflives

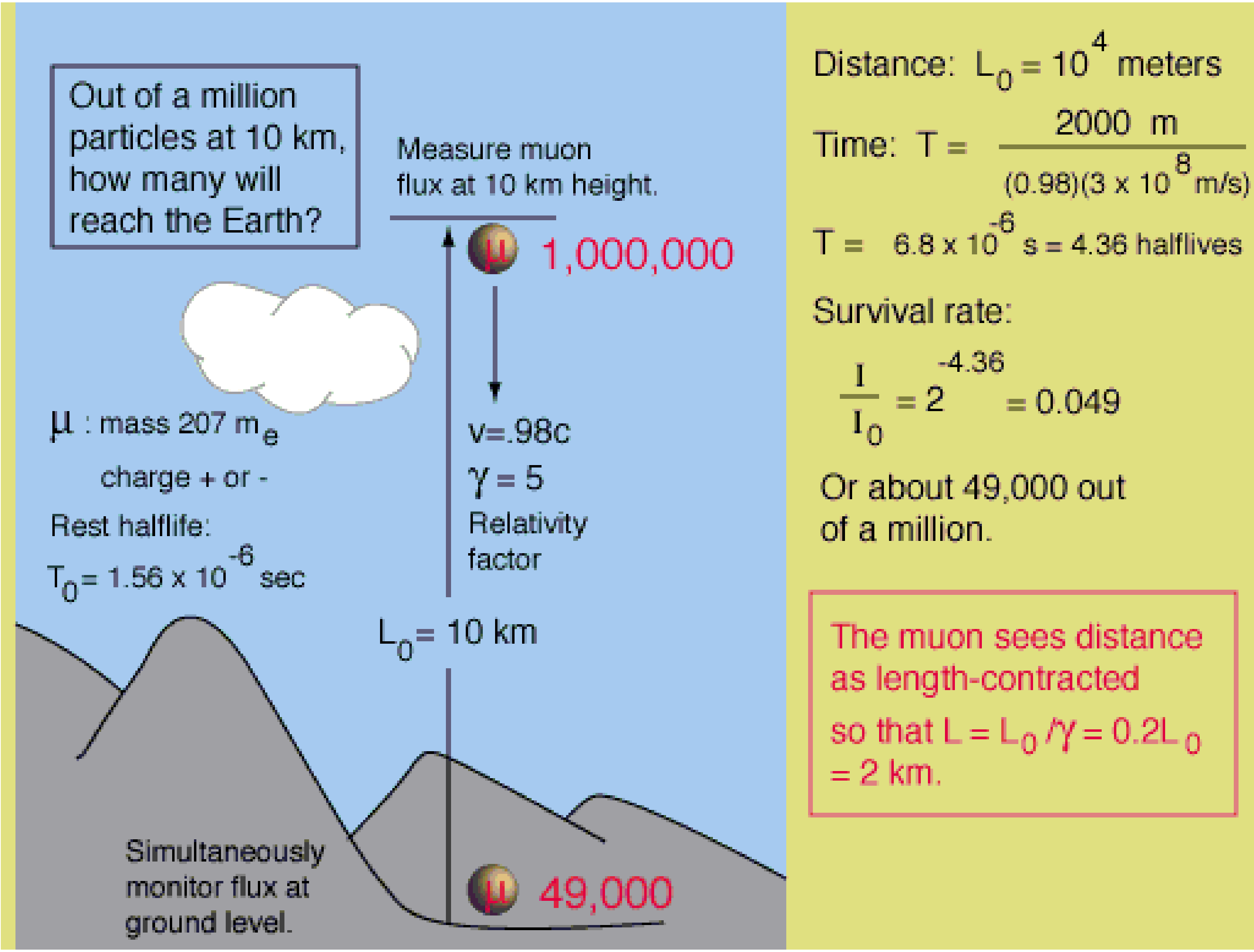
Survival rate:

$$\frac{I}{I_0} = 2^{-4.36} = 0.049$$

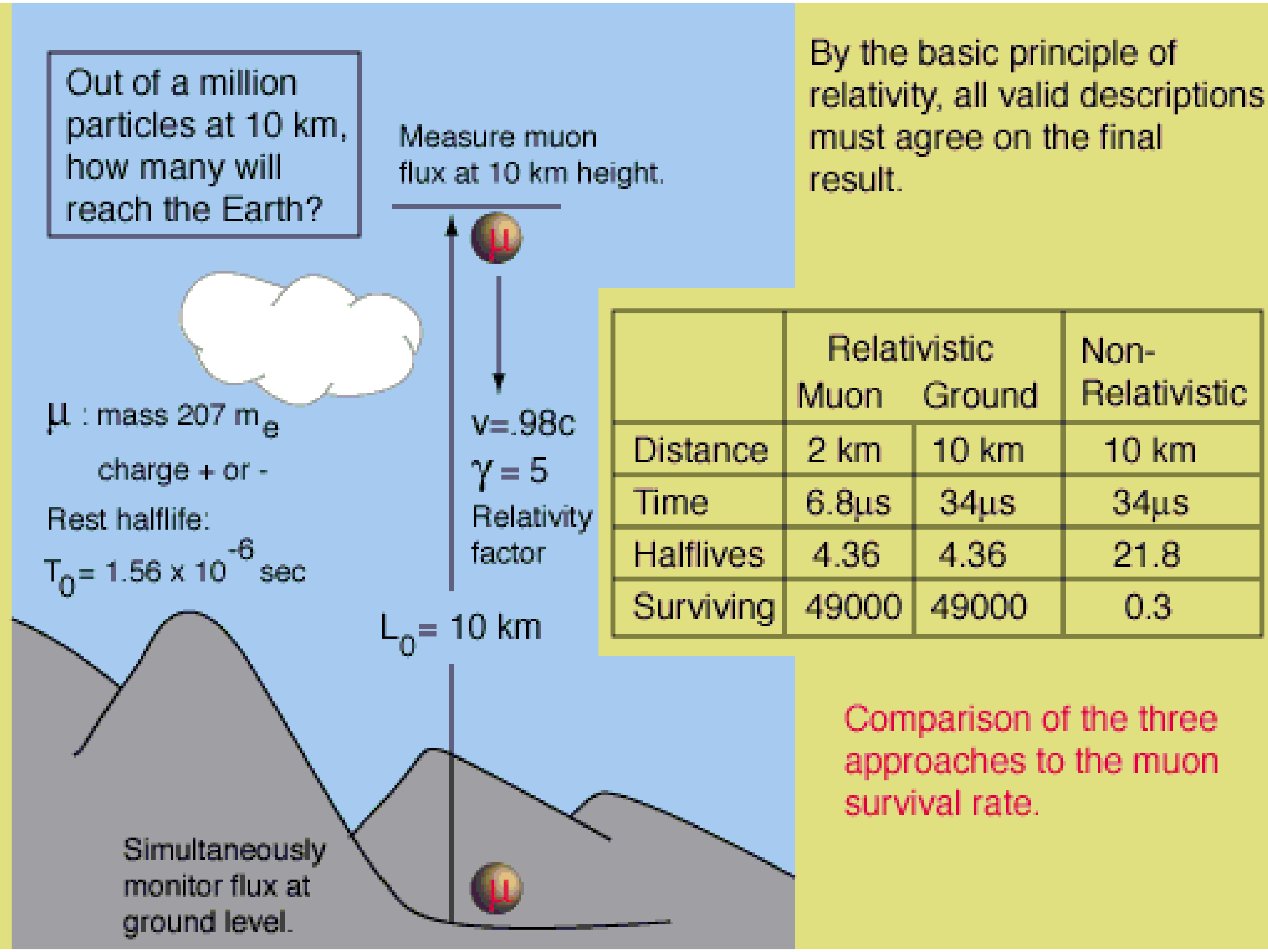
Or about 49,000 out of a million.

The muon's clock is time-dilated, or running slow by the factor $T = \gamma T_0$, so its measured halflife is $5 \times 1.56\mu s = 7.8\mu s$.

Relativistic, Muon-Frame Observer



Comparison of Reference Frames



	Relativistic Muon	Relativistic Ground	Non-Relativistic
Distance	2 km	10 km	10 km
Time	6.8μs	34μs	34μs
Halflives	4.36	4.36	21.8
Surviving	49000	49000	0.3

In the muon experiment, the relativistic approach yields agreement with experiment and is greatly different from the non-relativistic result. **Note that the muon and ground frames do not agree on the distance and time, but they agree on the final result.** One observer sees time dilation, the other sees length contraction, but neither sees both.