From the Hot Big Bang to the Concordance Model

- Cosmological components
    - Baryonic matter
    - Dark matter
    - Dark energy
- Inflation
- Structure formation
- Concordance Model of Cosmology

- **Lecture 2:**
    - Refining the Concordance Model (running primordial spectral index, neutrinos)
    - Cosmological probes

Describing the Universe

Golden rules:

1. A cosmological model should agree with observations.

2. Any parameter should be motivated by theory.

Success of the Hot Big Bang model

- Predicts evolution of the universe from a very hot and dense initial state.
- Explains observed expansion of the universe (Hubble relation).
- Provides conditions for nucleosynthesis: As universe expands and cools down, particles start to form, then nuclei, atoms...
- Excellent agreement with requirements from Particle Physics.

Cosmological components: **baryonic matter** Ω_b

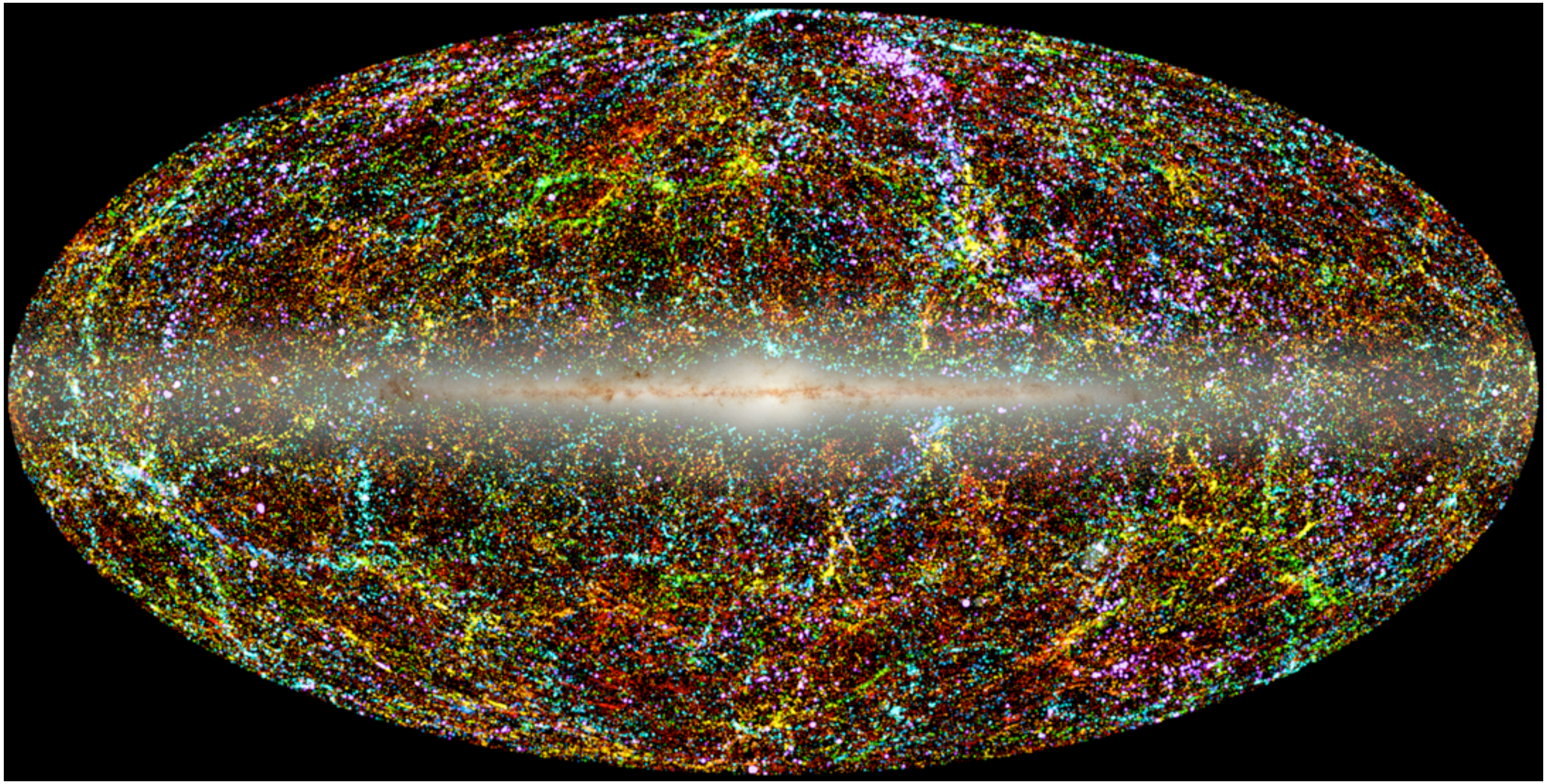
- Ordinary matter
- Interacts with photons (i.e. visible)

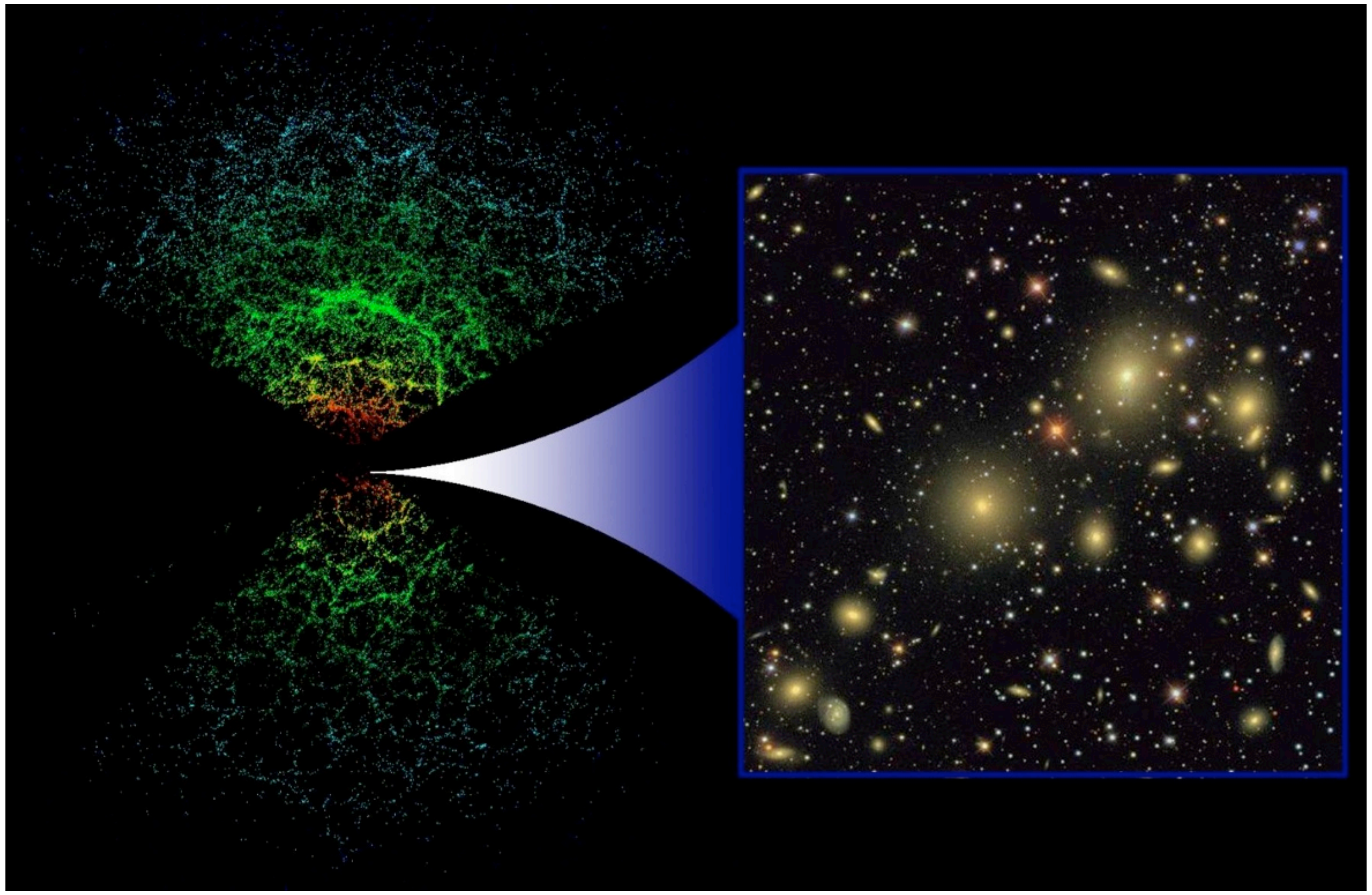
For cosmologists: **Baryons = any particle in the Standard Model that interacts with gravity**

- Density pinned down by a variety of arguments (primordial nucleosynthesis, D and He abundances, ...)
- Agreement between predictions of Standard Model of particle physics and astrophysical observations

- Involved in structure formation (interacts with gravitation)
- Makes up 5% of the Universe
- ~ 0.3 baryons per cubic metre

- Cosmic baryons: stellar matter (c. 1%) + intergalactic material (c. 4%)
  - Can be **luminous** (stars)
  - or **non-luminous**
  - non-emitting gas, compact objects: black holes, neutron stars, brown dwarfs
  - All these objects have been observed





Observations of distribution of baryonic matter in the Universe

BUT:
Do baryons make up the TOTAL matter content?

$$\Omega_b = \Omega_m \ ?$$

No

Cosmological components: **Dark matter**

- **How do we weigh astrophysical objects?**
  - Mass-luminosity ratio M/L
    - Only force at work is gravity, so luminosity of an object depends on its mass
- Galaxy luminosity function; Press-Schechter function (1974)
- Therefore: **Observe luminosity -> infer mass**

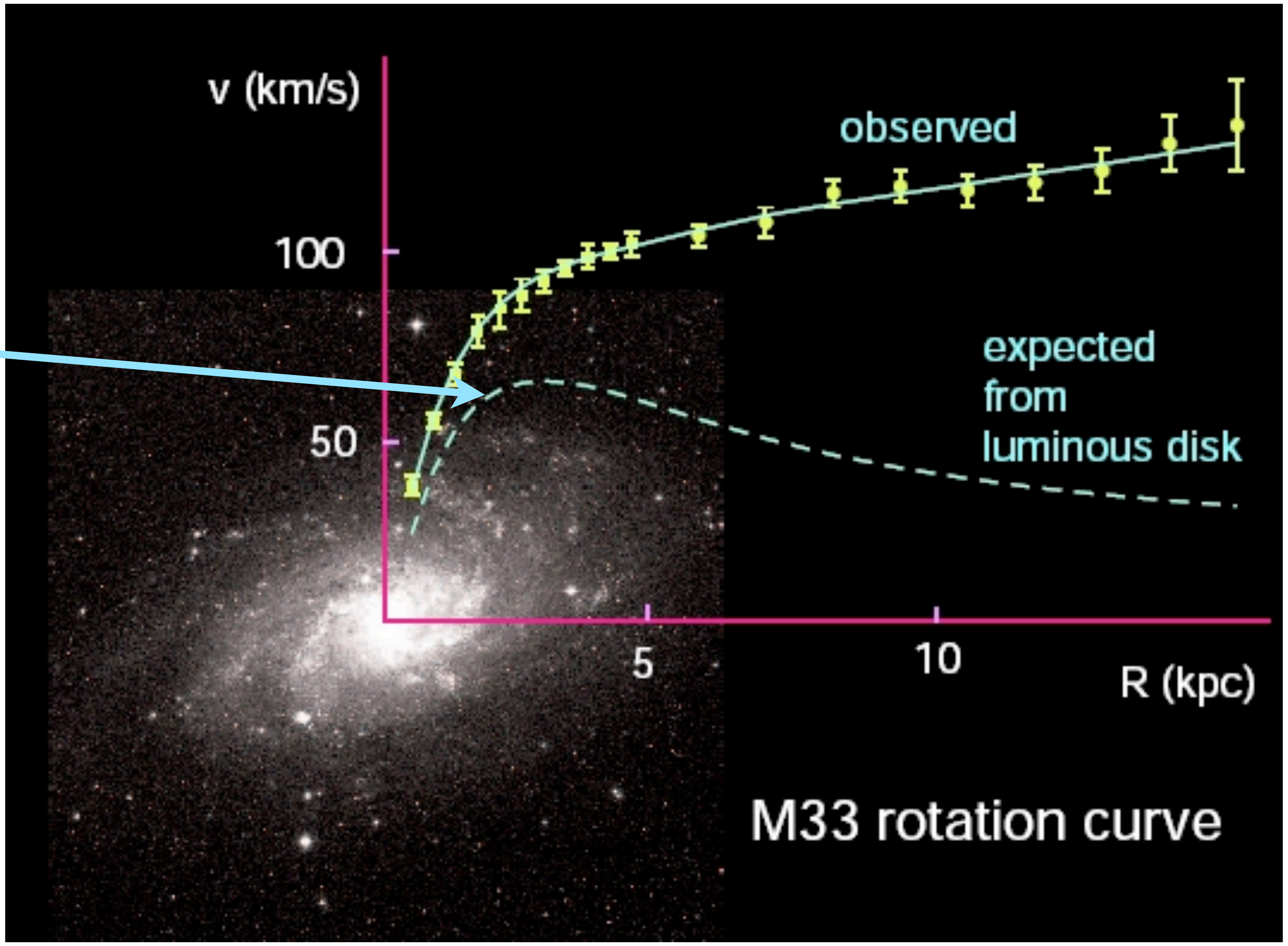
Evidence for 'hidden mass'

- Oort, Zwicky (1930s) - clusters of galaxies would be unbound without some hidden mass (dark matter)
- (1950s) - motion of Andromeda towards us implies hidden mass
- Dynamical evidence (1970s) rotational speeds of galaxies
  - orbital velocities of galaxy clusters

Kepler's laws: mass outside radius R contributes no gravitational pull

$$v = \sqrt{\frac{GM(R)}{R}}$$

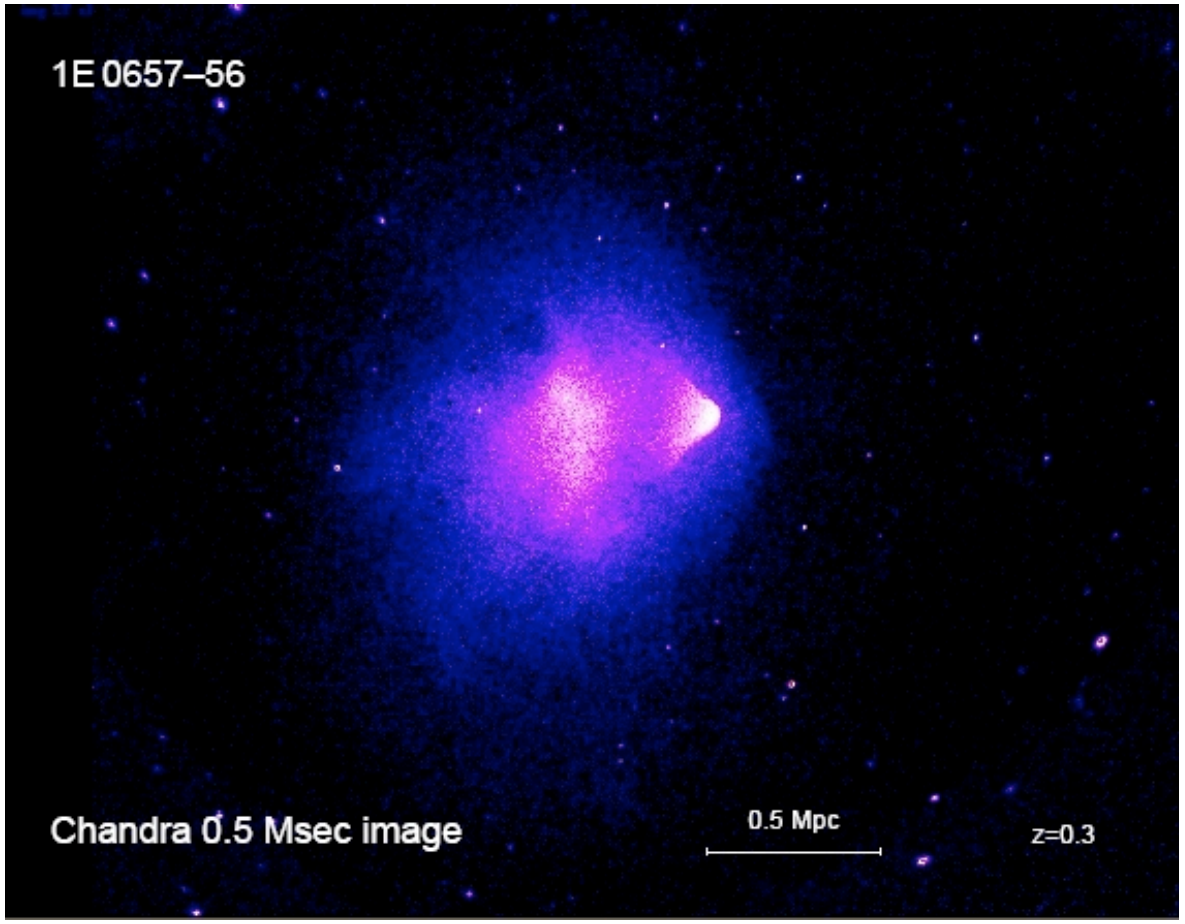
BUT: We observe roughly constant rotation curves at large radii
THEREFORE: **There is some mass outside the observed radius**

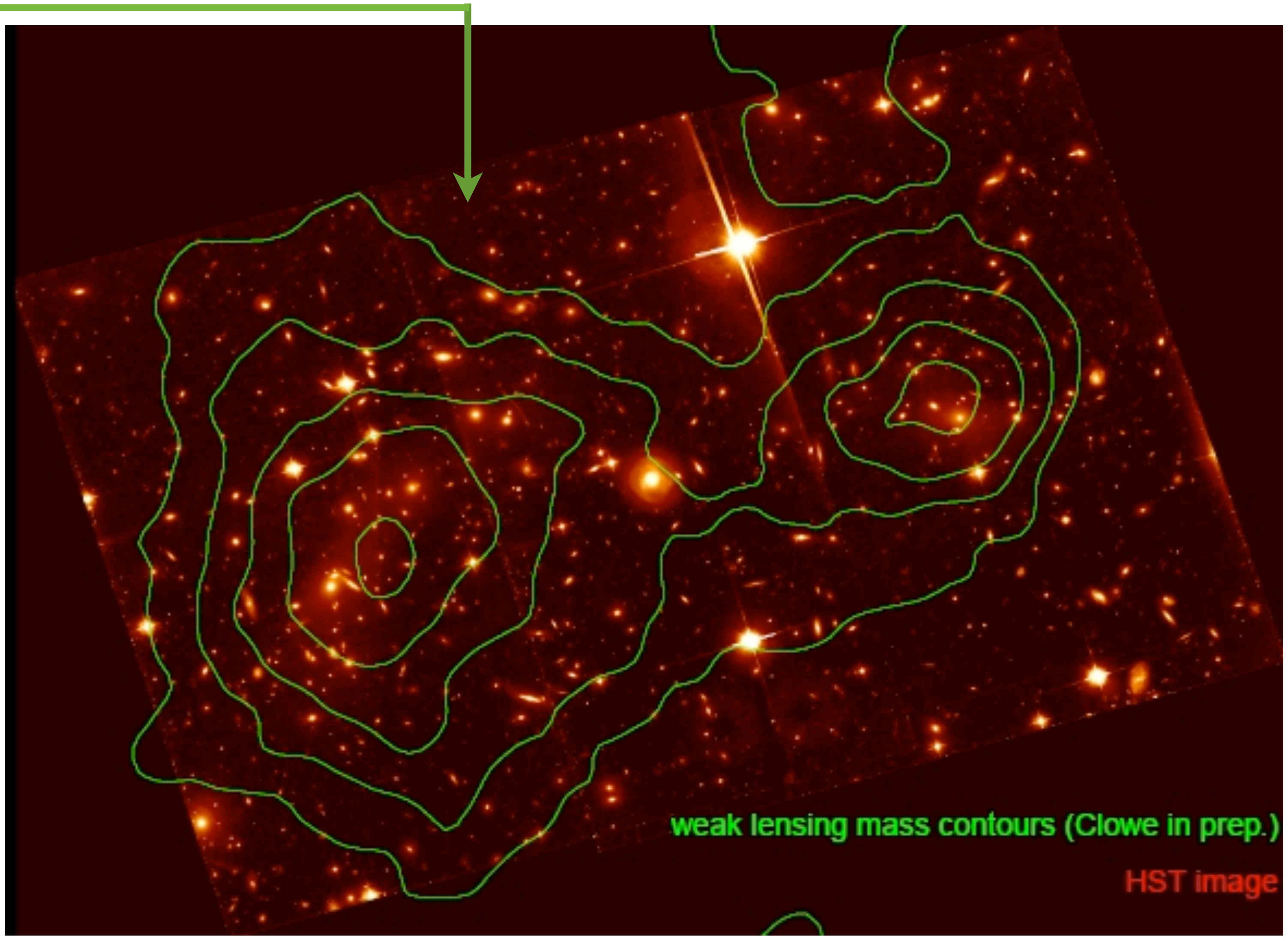


How does dark matter behave?

- **Evidence from the Bullet Cluster:** two galaxy clusters which have collided
- displacement of centre of mass from centre of baryonic mass cannot be explained by modified Newtonian dynamics (MOND). Requires dark matter
- Dark matter clump is coincident with the collisionless galaxies, but lies ahead of the collisional gas. This allows good limits on the cross-section of the self-interaction of dark matter.
- Only interaction with baryonic matter: gravitation
- Non-luminous component (Cold Dark Matter - CDM)

- Halos ('coronae') around galaxies





- **25% of the Universe is dark matter**
- Its distribution follows that of baryonic matter
- **Dominates** the formation of structure
- Galaxy formation involves at least **two** things
  - dark matter halos must form (relatively straightforward)
  - baryons must fall in and make stars (complex physics)
- Distribution of DM around galaxies is therefore modelled by semi-analytical fits

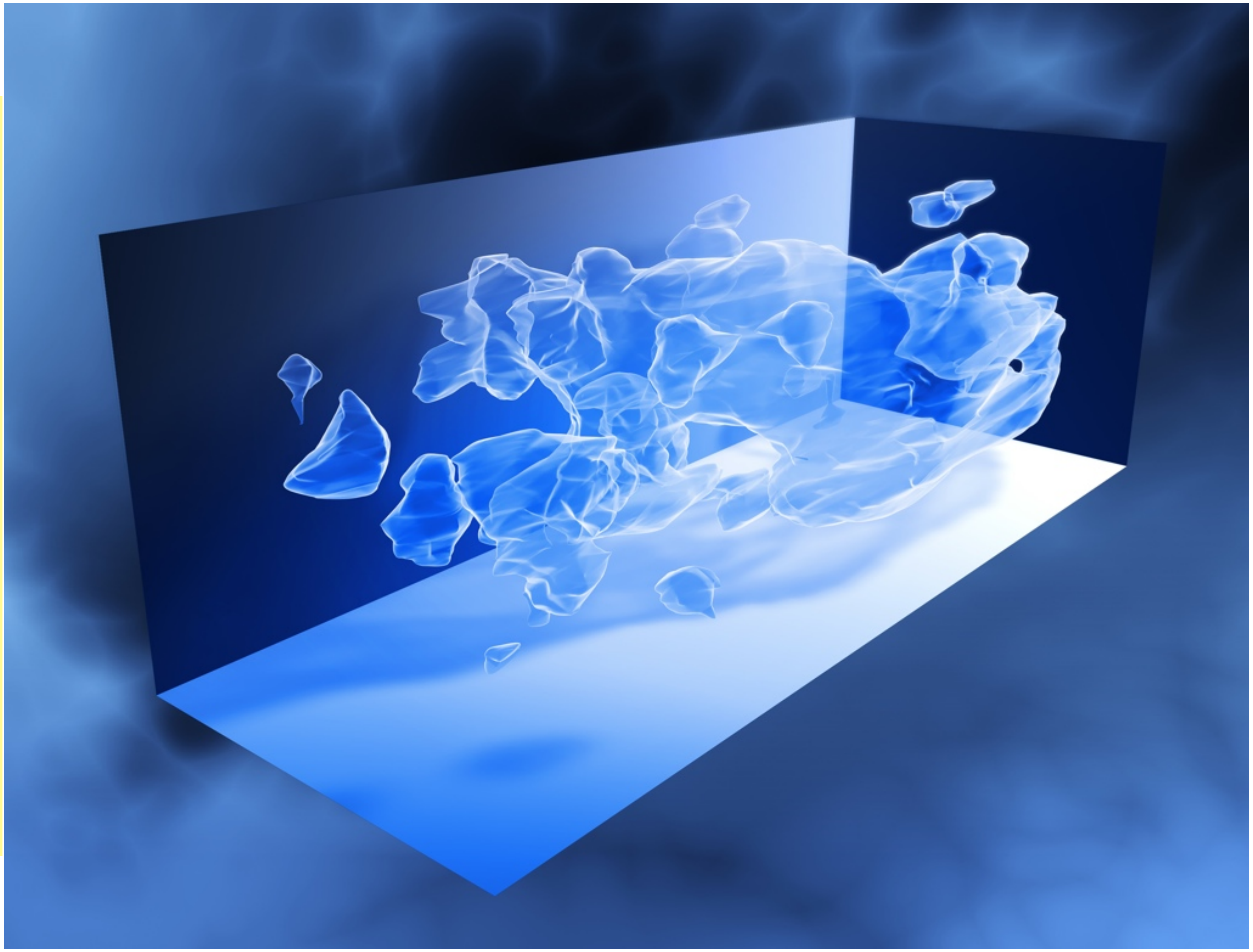
Dark matter candidates

Fundamental particles (?)

- Supersymmetric particles
- Sterile neutrino
- WIMPs (Weakly Interacting Massive Particles)
  - If so, direct detection?
  - BUT v. low cross-section

New physics (?)

- Modified gravity
- Interaction between dark matter and dark energy



Dark energy

Evidence for dark component:

- Based on the **Hubble rate**: redshift-distance relation, determined by energy content of the universe
- Observations of supernovae up to high z fix **Hubble rate** to unprecedented accuracy (Riess et al. 1998, Perlmutter et al. 1999)
- Further evidence: galaxy clusters

Conclusion 1: The expansion is accelerating

- Observations of CMB show only slight temperature differences across sky

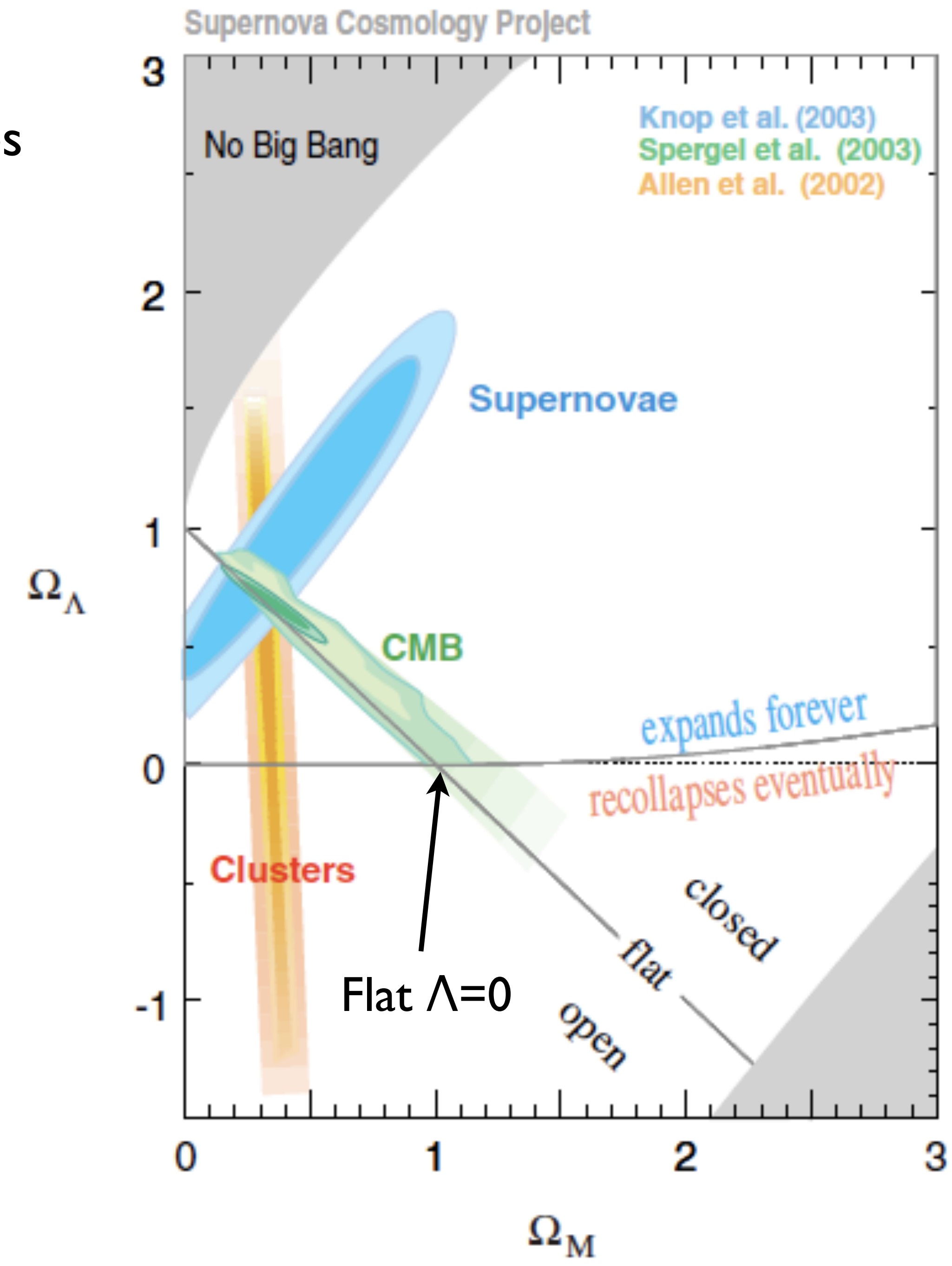
Conclusion 2: The Universe is flat

- **GR** is correct
- **FRW** metric is correct (homogeneous + isotropic Universe)
- Measurement of Hubble rate + geometry give us

$$\Omega_m = 0.3 \quad \text{but} \quad \Omega_{\rm tot} = 1$$

THEREFORE there is an additional 'dark' component such that

$$\Omega_{\rm tot} = \Omega_m + \Omega_{\rm DE}$$



Dark energy as a cosmological constant

- What if dark energy is the cosmological constant Lambda?
- Introduced by Einstein to balance curvature and obtain a static universe.

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

- Extra term in the Friedmann equation. Compatible with GR - does not change theory

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \longleftarrow$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}$$

- Acts as a repulsive force:

Although Λ is constant, its density is not, since H varies with time

$$\uparrow \frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \uparrow$$

- Can treat as fluid with energy density ρ and we obtain the fluid equation for DE:

$$\dot{\rho}_{\mathrm{DE}} + 3\frac{\dot{a}}{a}\left(\rho_{\mathrm{DE}} + \frac{p_{\mathrm{DE}}}{c^2}\right) = 0$$

Equation of state for DE fluid:

$$w = \frac{p_{\mathrm{DE}}}{\rho_{\mathrm{DE}} c^2}$$

Fluid has a **negative** pressure

For a cosmological constant $w = -1$

We have acceleration if w<-1/3

Reminder

dust: w=0 radiation: w=1/3

Dynamical dark energy (and other possibilities)

If dark energy is a fluid, its equation of state could exhibit slow variation (quintessence)

Chevallier - Polarski - Linder parameterisation (2001, 2003):

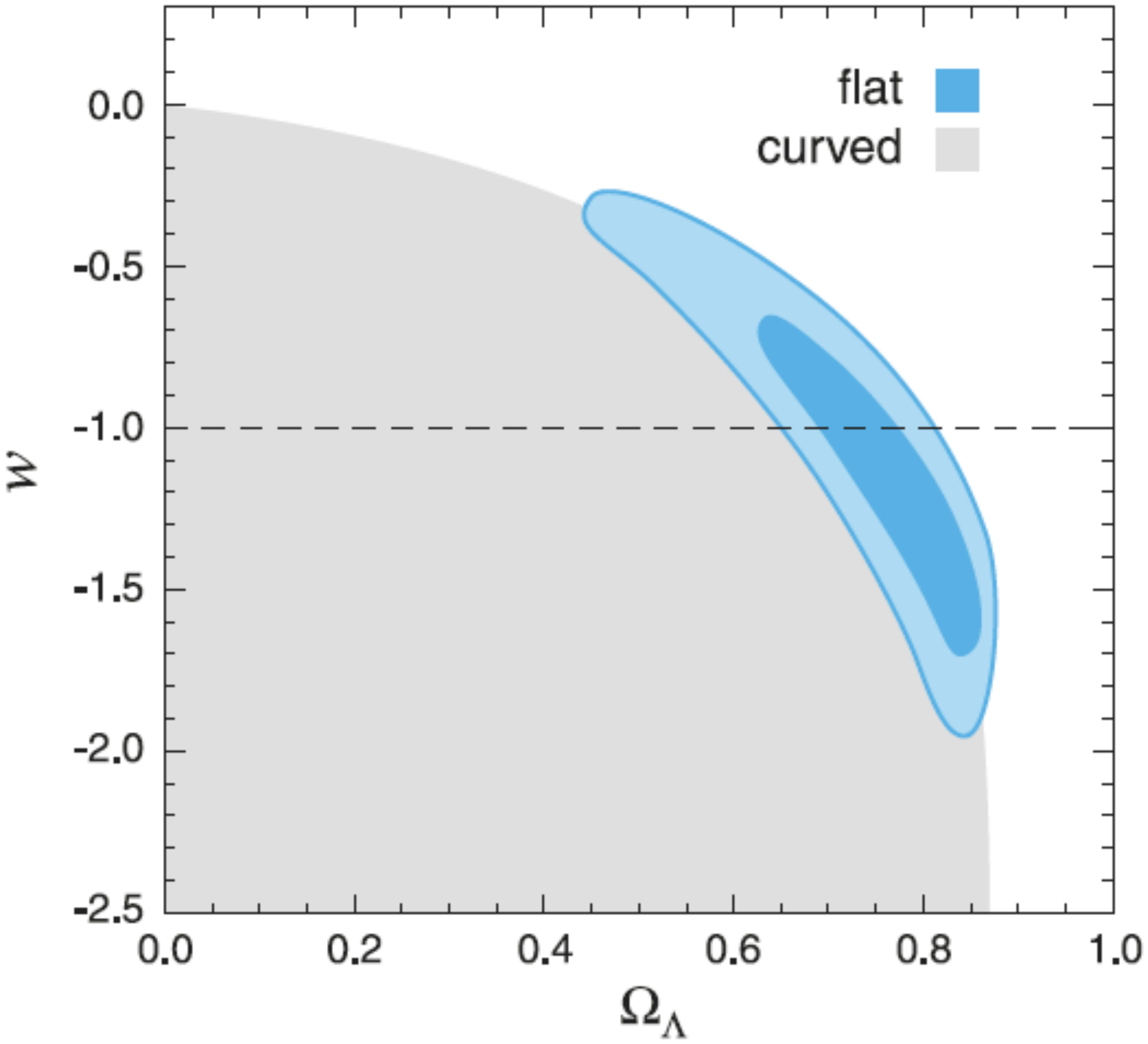
- Equation of state may depend on redshift
- If w_a=0 then w(a) is a constant

$$w(a) = w_0 + (1 - a)w_a$$

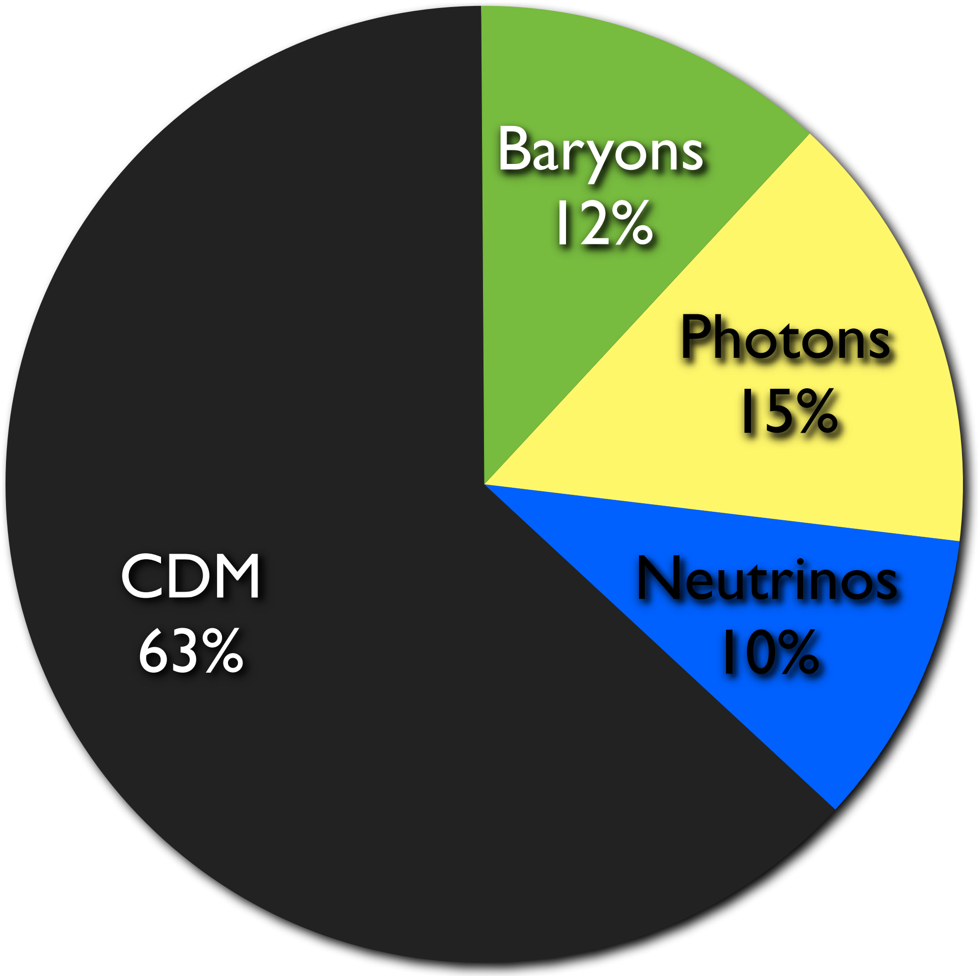
$$a = 1/(1+z)$$

Dark energy candidates

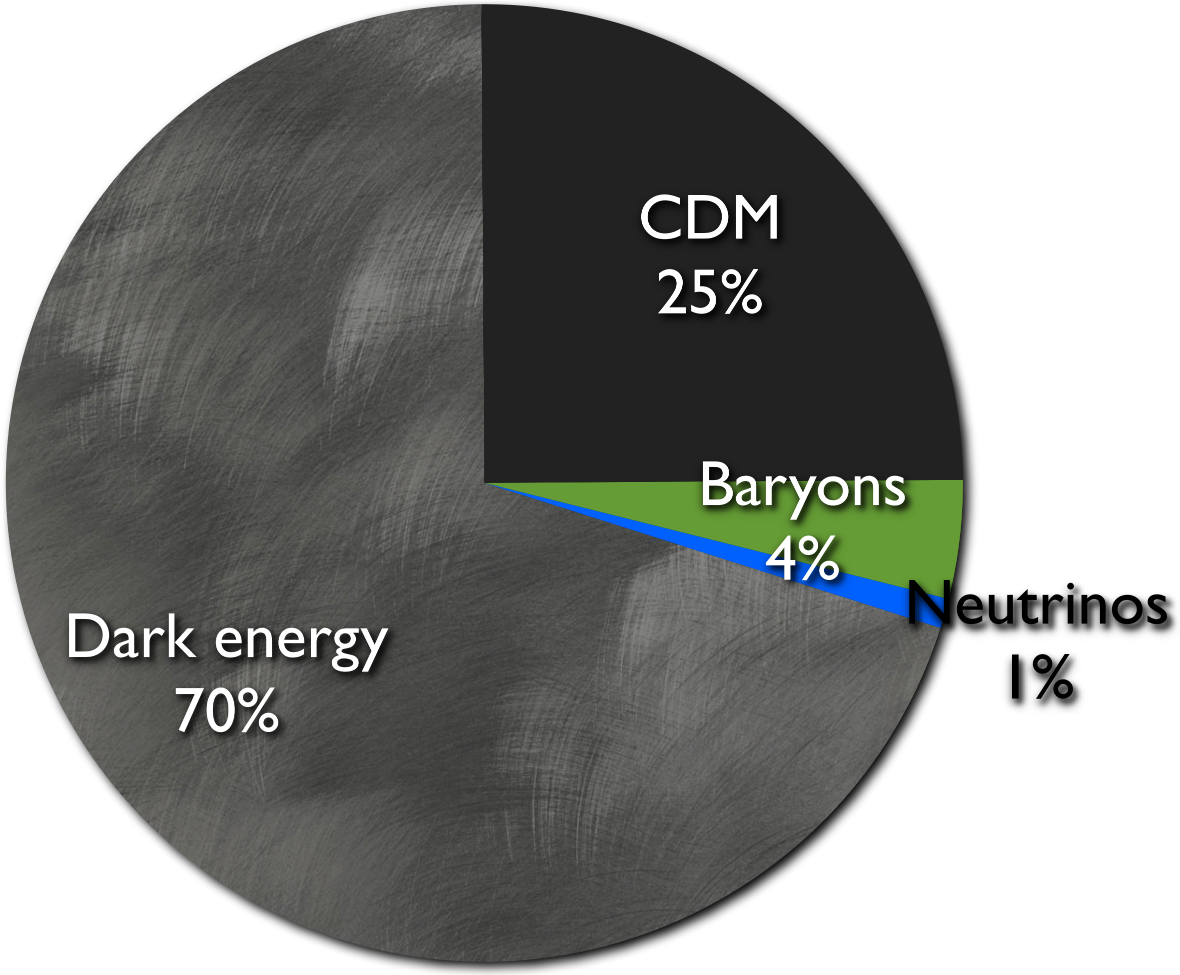
- w is still poorly constrained. Cannot rule out varying eqn. of state.
- Cosmological constant Λ compatible with GR
  - Could it be the vacuum energy?
  - Fine-tuning problem: Too large by 120 orders of magnitude
    - Re-calculation of the value?
- Dynamical models (Quintessence)
  - Scalar field (New physics or can we use Standard Model?)



Energy budget of the Universe



13.7 billion years ago

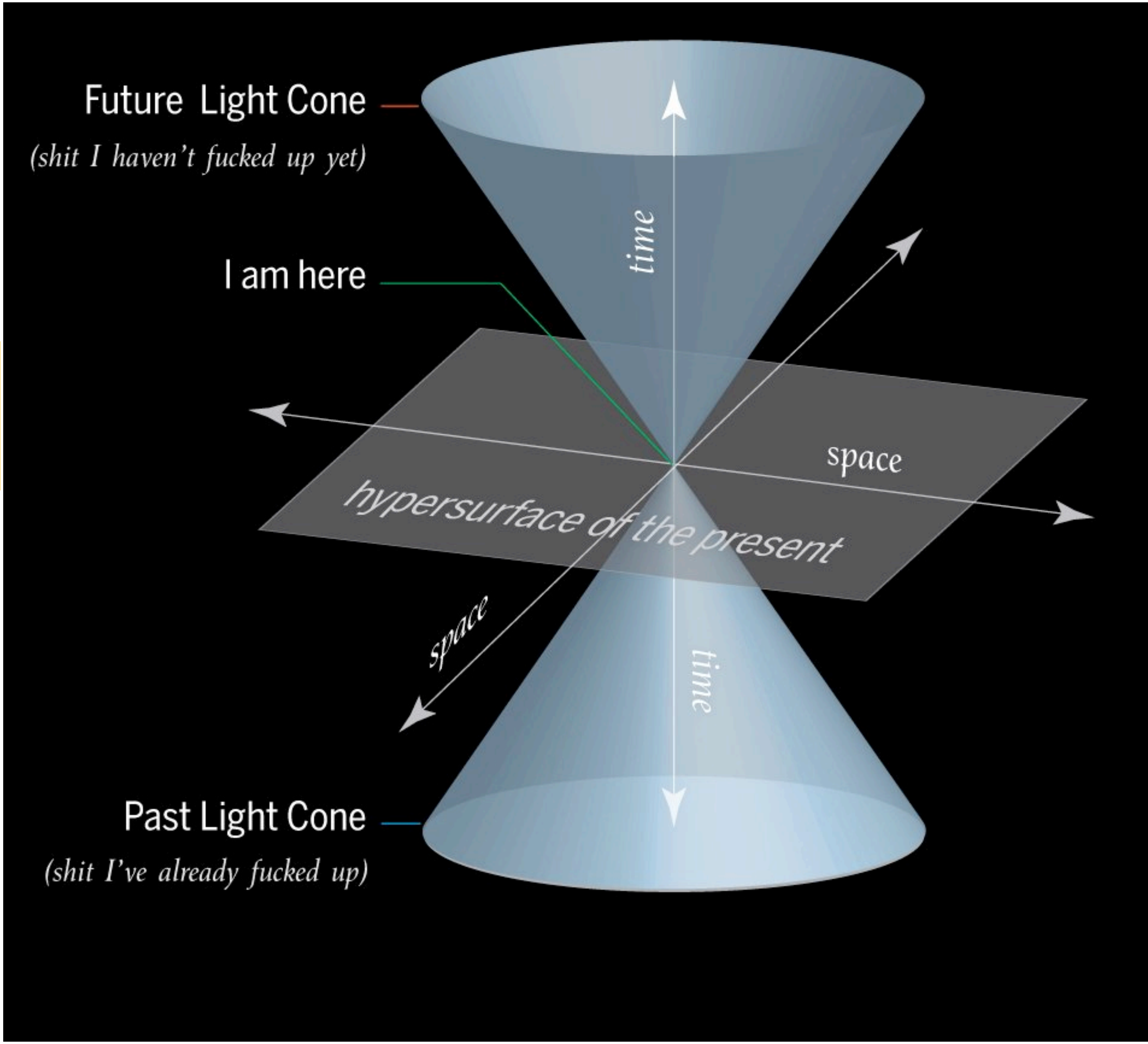


Today

Does the Hot Big Bang model with baryons + dark matter + dark energy give us the whole picture?

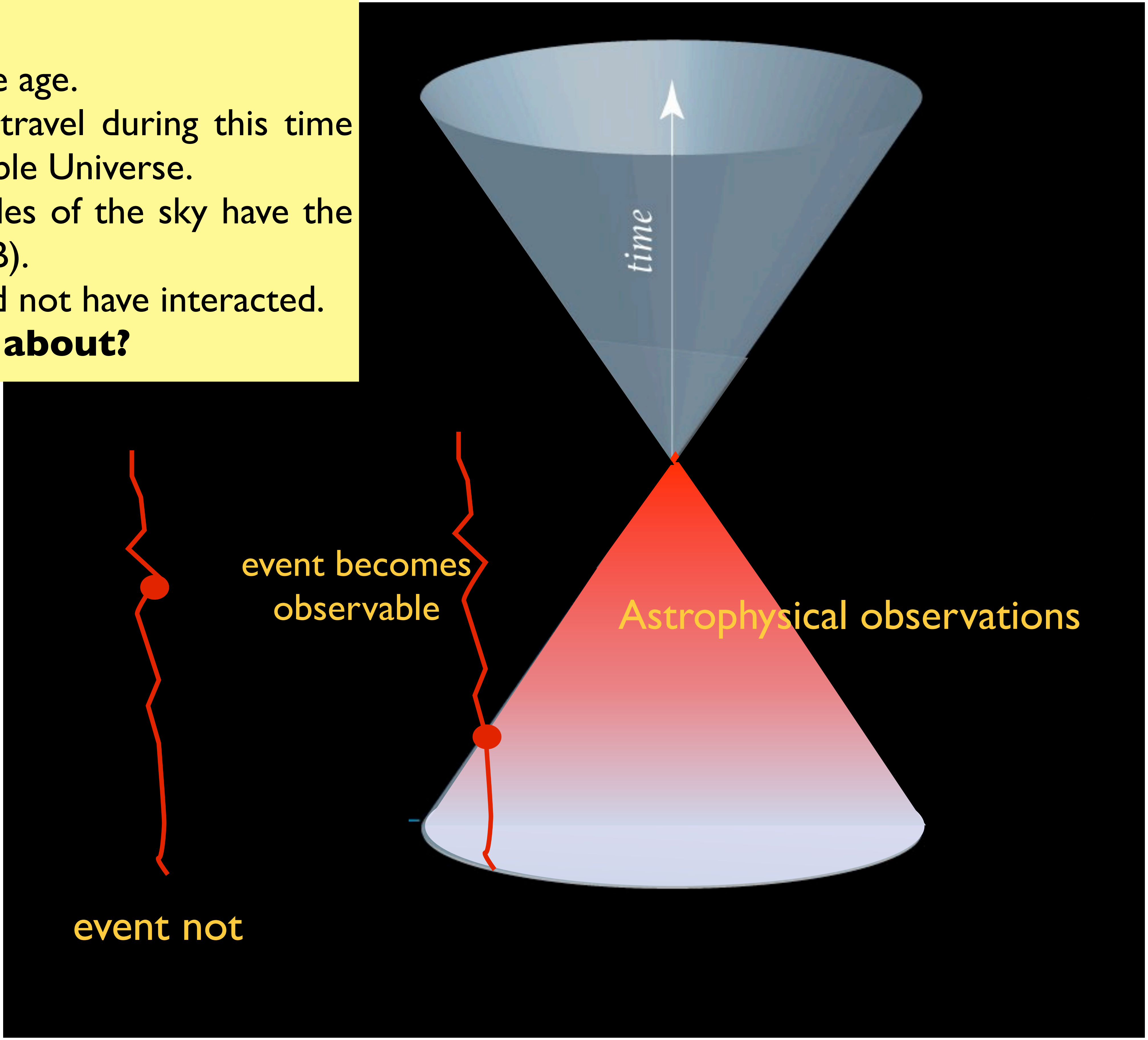
Problems with Hot Big Bang model

Recall the light cone



1. Horizon problem

The Universe has a finite age.
Distance that light can travel during this time determines the observable Universe.
Regions on opposite sides of the sky have the same temperature (CMB).
BUT these regions could not have interacted.
How can this come about?



2. Flatness problem

$$|\Omega_{\rm tot} - 1| = \frac{|k|}{a^2 H^2}$$

If $\Omega_{\rm tot} = 1$ then it remains so at all time.
BUT this is an unstable solution: any deviation from this value and the Universe will quickly become curved.
Why does the curvature have this value?

3. Monopoles problem

Grand Unified Theories predict the formation of a large number of magnetic monopoles.
BUT we do not observe them.

Structure formation problem

CMB shows slight fluctuations
Represent seeds of structure.
Big Bang model does not explain the formation of structure.
Model is inherently incomplete

Inflation

The solution to all these problems is **inflation** (Alan Guth 1981)

A period in the evolution of the Universe during which the scale factor was accelerating.

$$\text{Inflation} \iff \ddot{a}(t) > 0$$

- Idea is to extend Standard Model by changing the evolution of primordial phase to address the problem of initial conditions.
- **Main historical motivation: to resolve flatness problem.**
- Inflationary epoch lasted less than a millisecond.
- Expansion is perfectly exponential (scale factor increases exponentially).

Solving the Big Bang problems:

1. **Flatness:**

$$|\Omega_{\text{tot}} - 1| = \frac{|k|}{a^2 H^2}$$

$$\ddot{a} > 0 \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}(\dot{a}) > 0 \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}(aH) > 0$$

- LHS is driven to zero.
- Result: **Inflation drives universe to spatial flatness**.

2. **Horizon:**

- Small initial thermalised regions get blown up to encompass our entire observable Universe.
- Result: **We observe regions in thermal equilibrium.**

3. **Monopoles:**

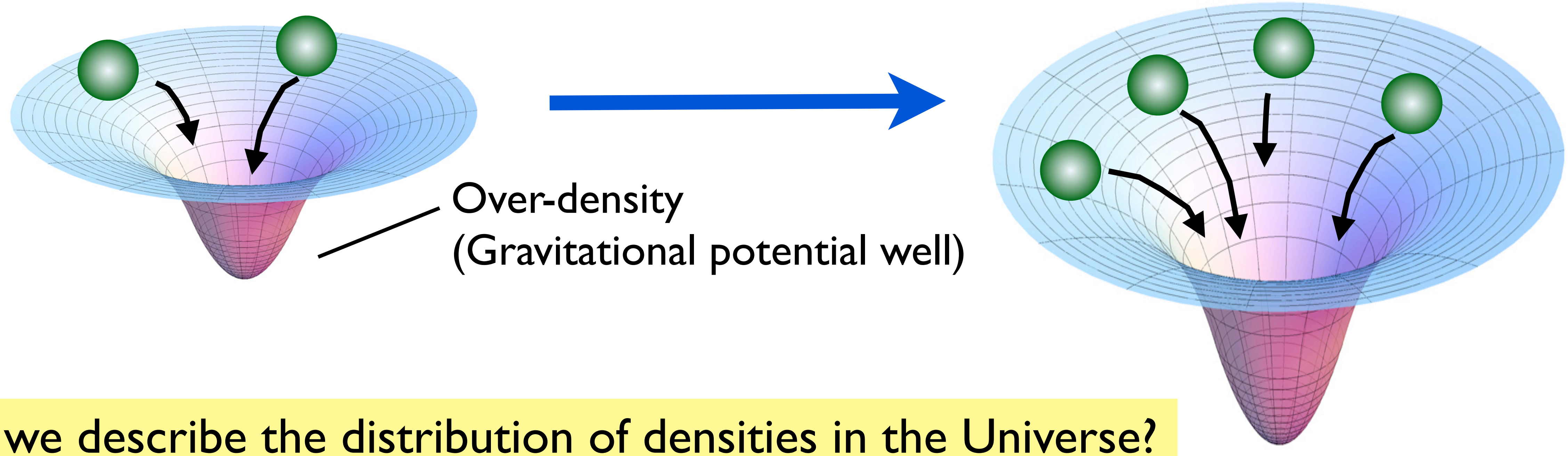
- Expansion during inflation dilutes away any relic particles.

Inflation and initial conditions

- The inflationary scenario provides an explanation for the **growth of structure**.
- Most models of inflation are **slow-roll**, in which Hubble rate varies slowly.
- Inflation caused by an ‘inflaton field’ with some potential.
- Different points in the universe inflate from different points in the potential.
- Inflation for these points therefore ends at different times.
- Result: This induces a **density fluctuation**....
- .... which provides the initial seed for **structure formation**.

Structure formation

- One obvious fact: **The Universe is not completely homogeneous.**
- Matter collapses **gravitationally** around initial mass over-densities. Effect is then **amplified**.



How do we describe the distribution of densities in the Universe?

Density at point x

Relative density →
$$\delta(x,a) = \frac{\rho(x,a) - \overline{\rho}(a)}{\overline{\rho}(a)}$$
Mean density

$$\langle \delta(x) \delta^*(y) \rangle = C_{\delta\delta}(|x-y|)$$

Correlation function:
Expression for **density field**

Fourier transform:
$$\langle \hat{\delta}(\mathbf{k}) \hat{\delta}^*(\mathbf{k}') \rangle = \int \mathrm{d}^3 x e^{i\mathbf{k}.\mathbf{x}} \int \mathrm{d}^3 x' \mathrm{e}^{-i\mathbf{k}'.\mathbf{x}'} \langle \delta(\mathbf{x}) \delta^*(\mathbf{x}') \rangle$$

Obtain expression for **matter power spectrum**

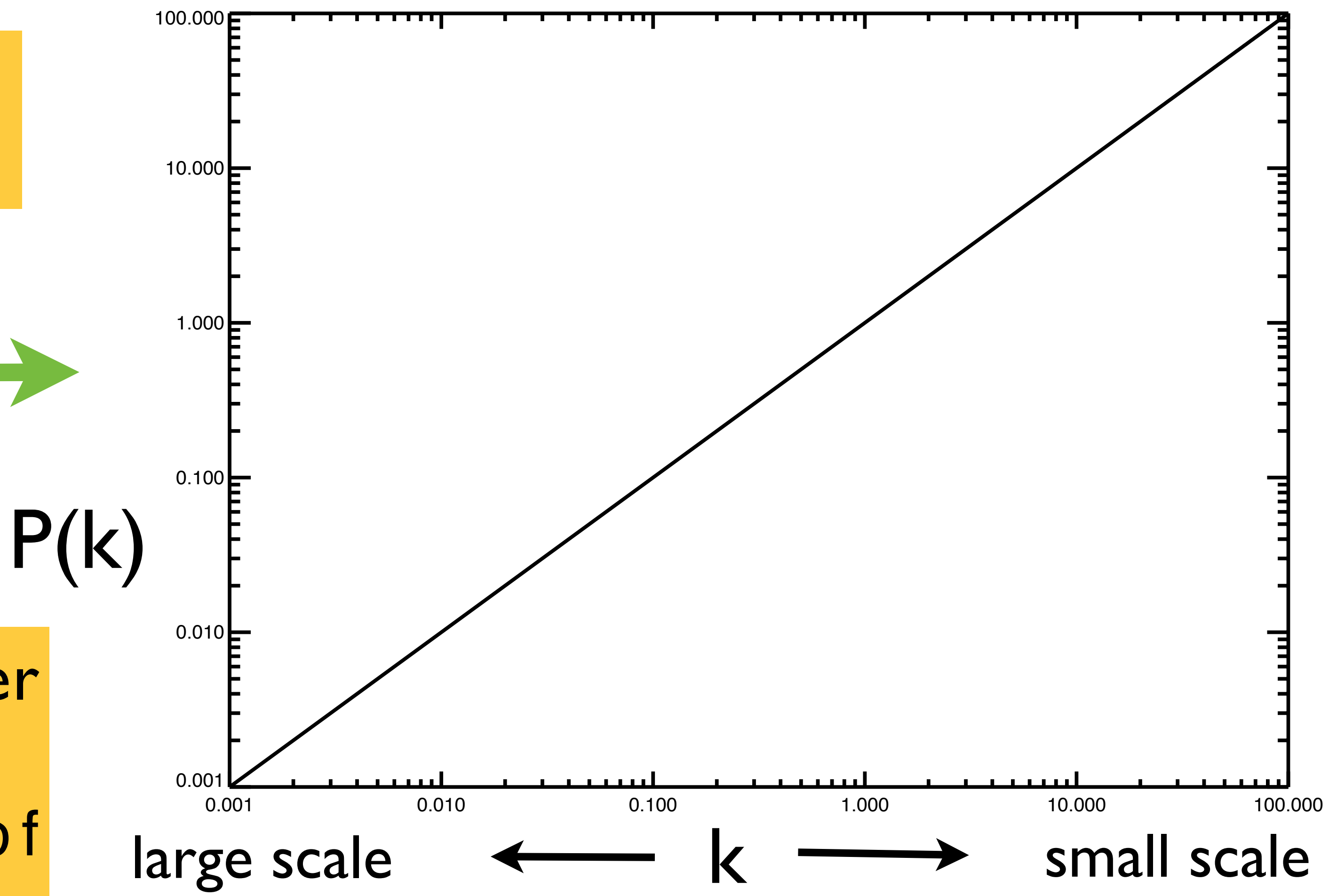
$$P_\delta(|\mathbf{k}|) = \int \mathrm{d}^3 y e^{i\mathbf{k}.(\mathbf{y})} C_{\delta\delta}(|\mathbf{y}|)$$

Usually written as P(k), where k indicates the scale

How do we calculate the Matter power spectrum?

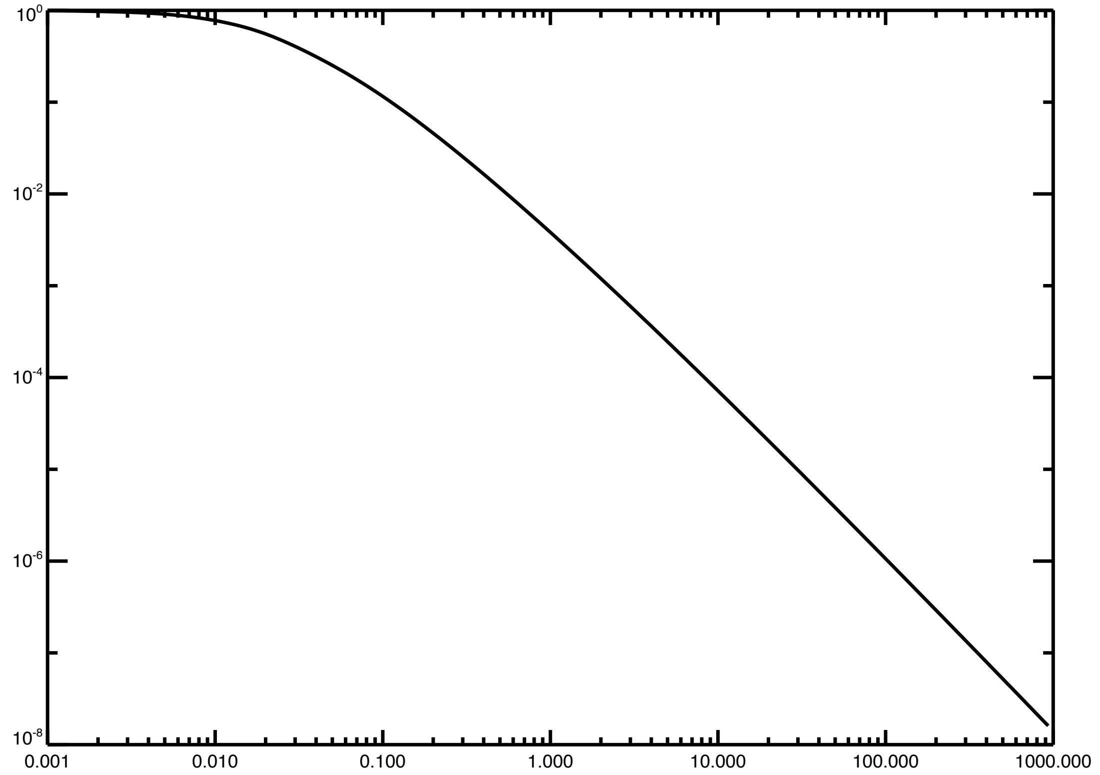
1. Start with some initial conditions:
Primordial power spectrum

Density fluctuations at the end of inflationary epoch →

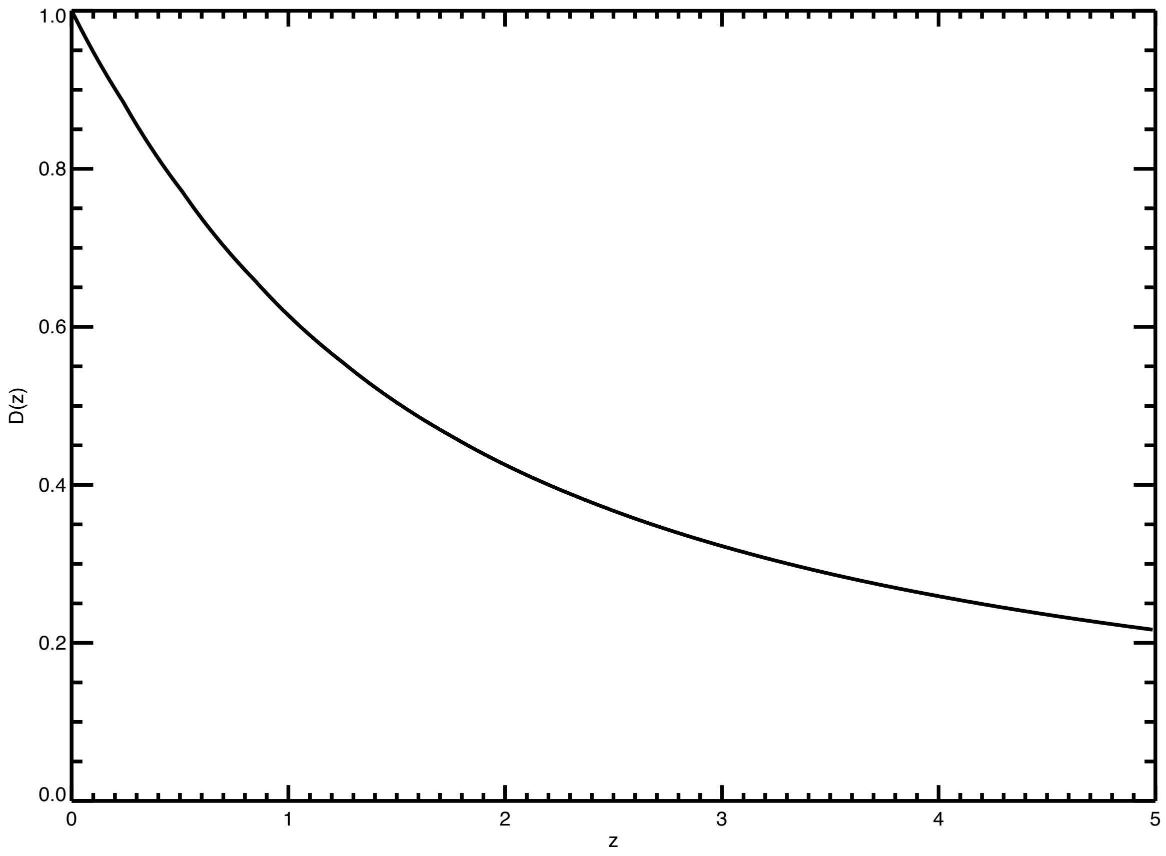


2. Model the behaviour of matter components: **Transfer function**

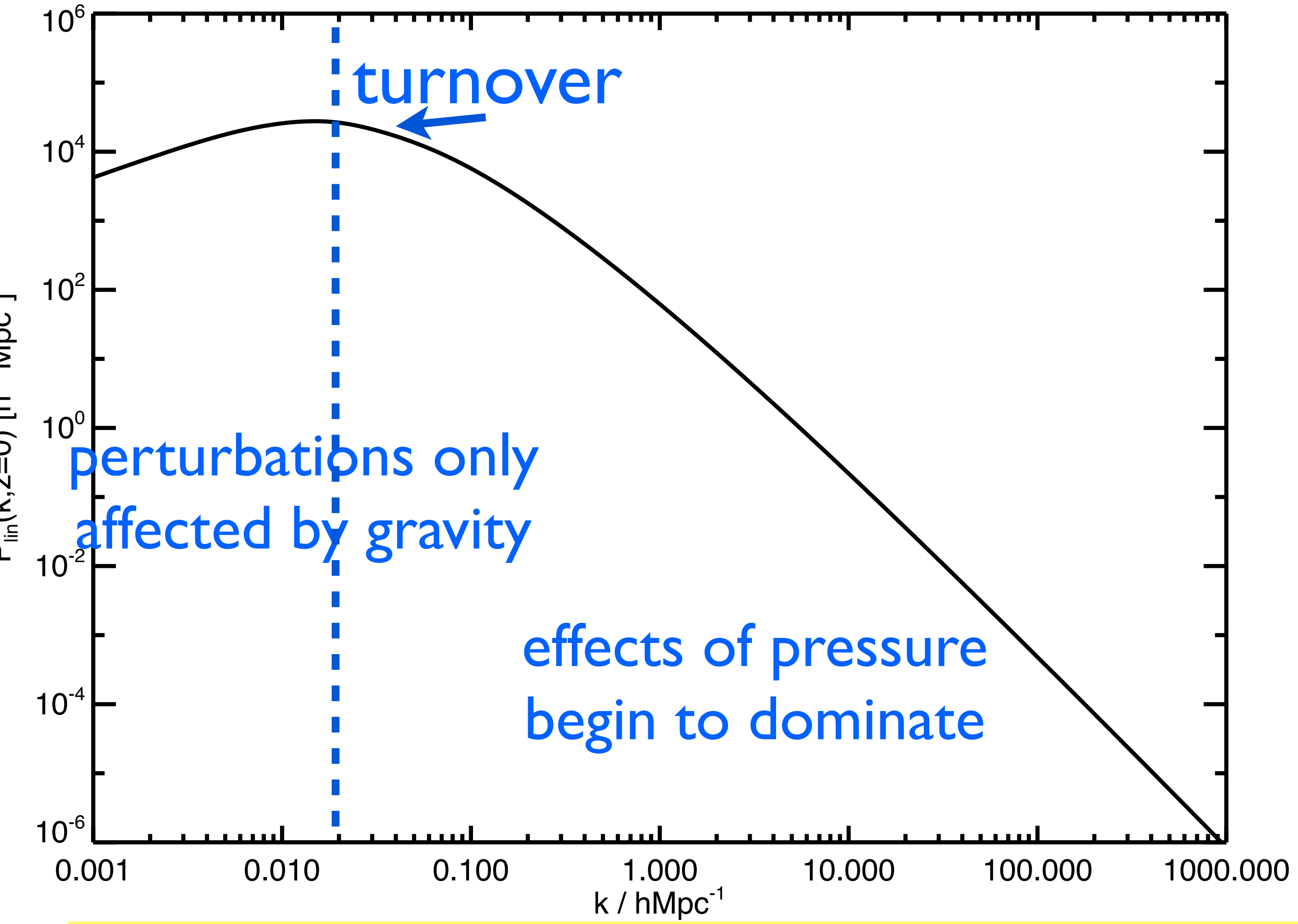
- Describes the evolution of perturbations during horizon crossing and the matter-radiation transition.
- Horizon = distance travelled by light since initial singularity
- Horizon-crossing = ‘size’ of a perturbation becomes as large as the horizon
- Horizon size determines turnover point



3. Model the evolution of the density field **at late times**, including the effect of dark energy, expansion, geometry of the Universe: **Growth function**



4. Obtain **linear matter power spectrum $P_{lin}(k,z)$**

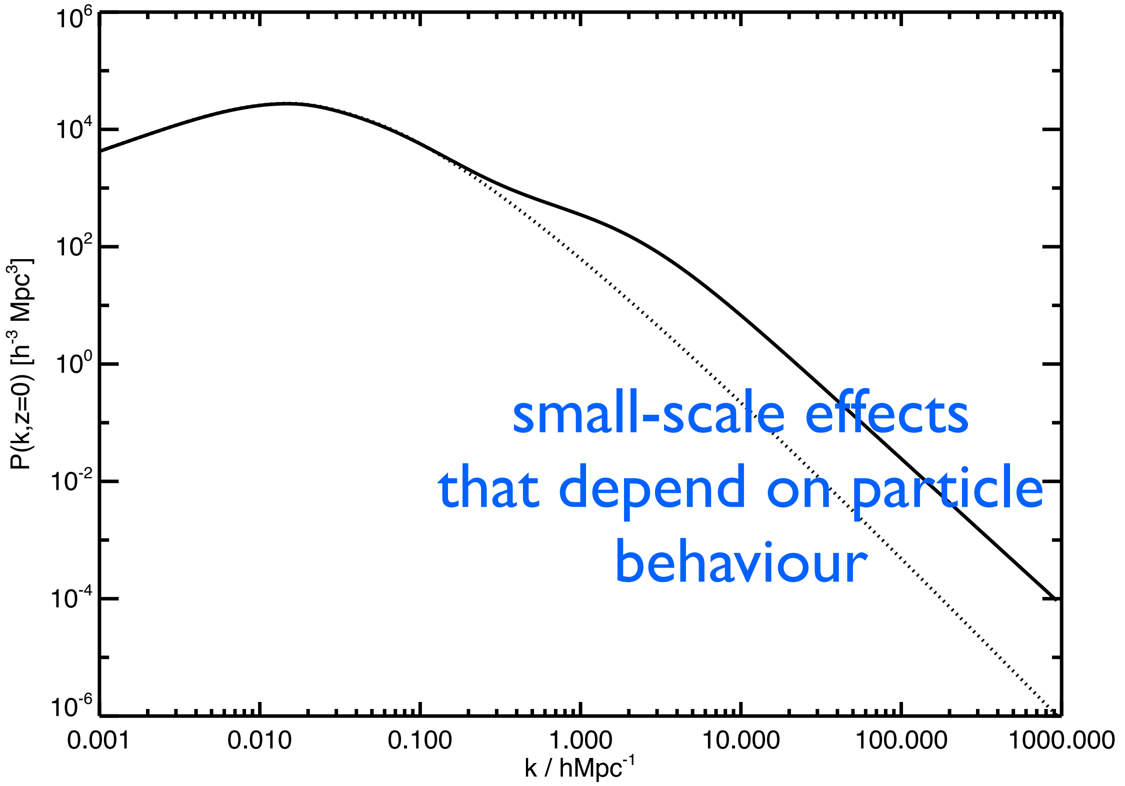


turnover

perturbations only affected by gravity

effects of pressure begin to dominate

5. Include non-linear corrections and finally obtain **non-linear power spectrum P(k,z)**

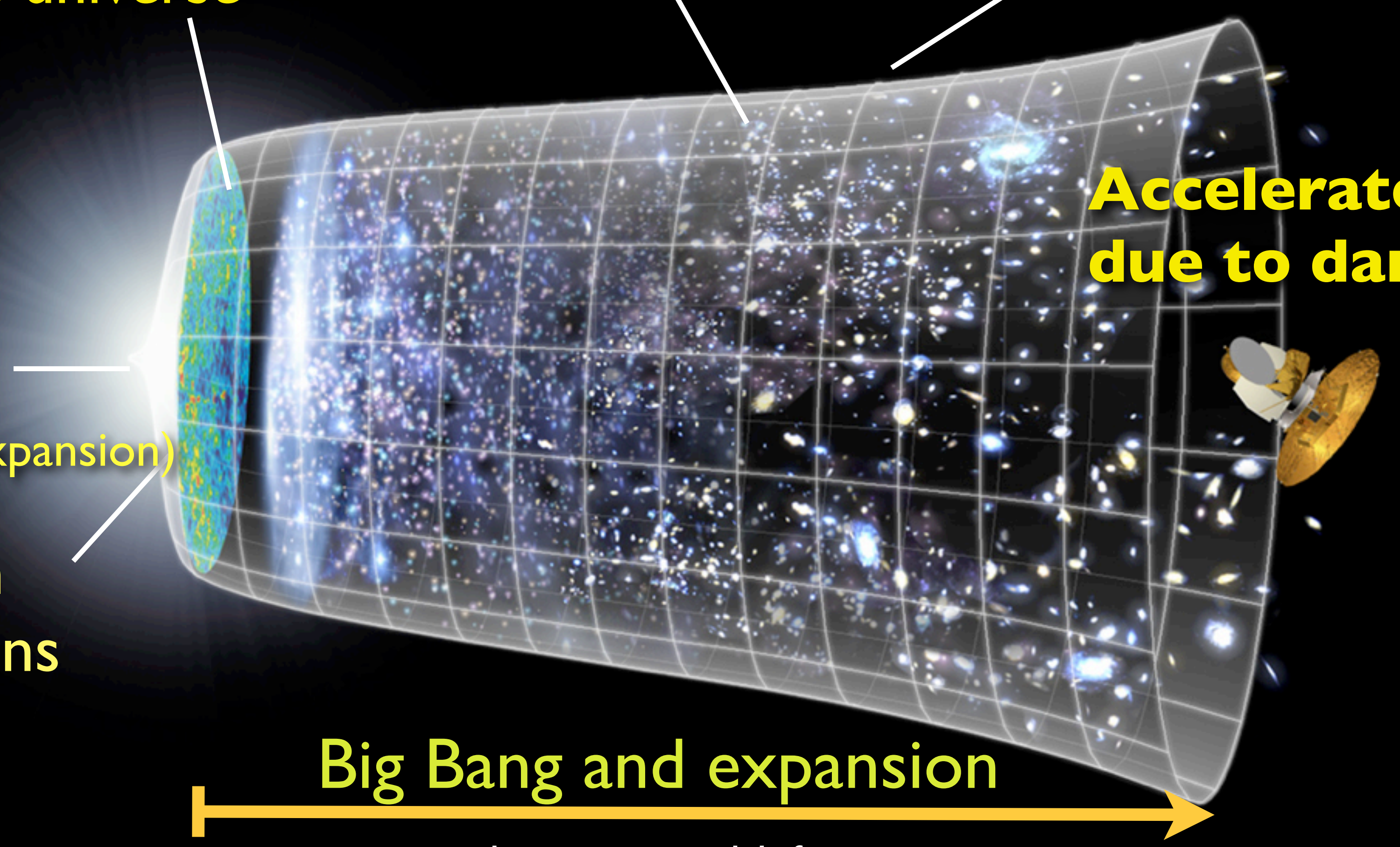


small-scale effects that depend on particle behaviour

$$P(k,z)= P_{prim}(k) \times T(k,z) \times G(z)$$

Conclusion: The Concordance Model

- Gravity described by General Relativity
- Initial conditions described by Inflation
- Content: Matter, radiation, dark energy



Distribution of matter in the Universe given by power spectrum

Large-scale structure formation

Initial conditions

homogeneous universe

Given by primordial power spectrum

Accelerated expansion due to dark energy

Inflation

(accelerated expansion)

Quantum fluctuations

Big Bang and expansion

decreasing redshift