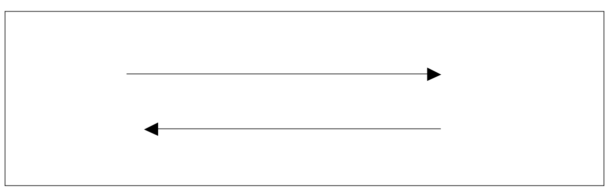
WAVES (cont'd)

INTERFERENCE - III

A particularly interesting case is when a wave is *confined* in a space (in 1, 2 or 3 dimensions).

Then, when the wave reflects off a boundary, it interferes with itself going the other way.



Using our usual method,

$$y = A\left[\cos\left(\omega t - kx\right) + \cos\left(\omega t + kx\right)\right]$$

which can be manipulated using the trigonometric identities as before, to give:

$$y = 2A\cos(\omega t)\cos(kx)$$

This is no longer a travelling wave $y = A\cos(\omega t - kx)$

Instead, each particle at position *x* undergoes a vibration with:

- frequency $w / 2p$

- amplitude that has a maximum value of 2*A* but is also modulated by a spatial function:

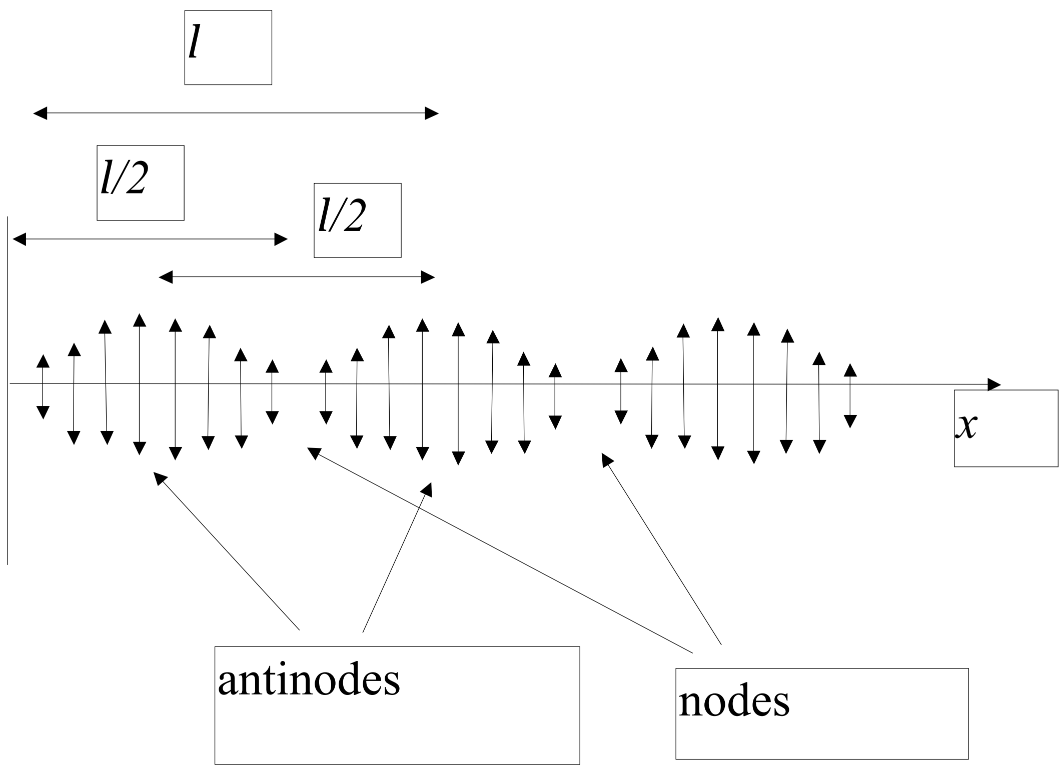
$$M = \cos kx = \cos \frac{2\pi}{\lambda} x$$

Thus, within the enclosure, there are positions where:

$M = \pm 1$ amplitude = 2*A* *x=0*, *l*/2, *l*, 3*l*/2,…
called antinodes

$M = 0$ amplitude = 0 *x*= *l*/4, 3*l*/4, 5*l*/4,…
called nodes

This wave-like spatial pattern does not change with time so the wave is called a *standing wave*

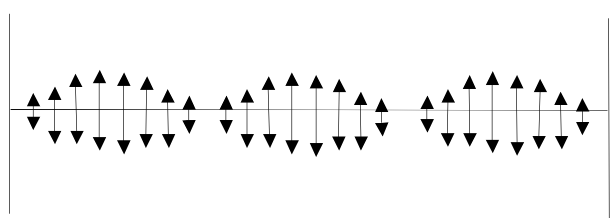


Since the $w\ t$ term in the equation does not contain any other term, the vibration of all particles is *in phase*: they are all at the extreme point of their motion at the same time, an instant later they are all at a corresponding intermediate position, and so on. There is even an instant when all particles are in their equilibrium position!

Notice in the above picture that the reflecting surface is a node: this must be so, since the particle "touching" this surface cannot move.

Of course, this applies to any other reflecting surface. In our 1-D example, therefore,:

> a standing wave can only exist between two reflecting surfaces when both surfaces can be nodes, ie. they are separated by a distance *nl/2*



(in the diagram, n=3)

If this condition is not met, the pattern of nodes and antinodes will move between the ends so the wave will be "smeared out" and not exist.

This has an interesting consequence. If l is the distance between the reflecting surfaces,

$$l = n\lambda / 2$$

$$v = f\lambda = f\,\frac{2l}{n}$$

$$f = n\frac{v}{2l}$$

so only certain frequencies can excite a standing wave. This phenomenon is called *resonance* and is a simple way of obtaining a pure frequency. It forms the basis of all musical instruments.

Musical instruments are very complicated vibrating systems and this is what gives them their distinctive tone.

Examples:

1-D oscillations: strings, air columns (organ pipes, etc).
NB: in an open pipe, the closed end is a node while the open end is an antinode

2-D oscillations: drum-skins, surfaces of guitar or violin

3-D oscillations: air vibrations inside kettle drums, violins, etc.

Note also:

- the above equation is satisfied for integral values of n which give rise to multiples or *harmonics* of the fundamental (lowest, $n=1$) frequency. Recall Fourier's theorem that harmonics make up the shape of the wave.

- When a system is excited at resonance, it vibrates, at other frequencies it does not. While this can be beneficial (eg: musical instruments), there are many cases where it is unwanted, especially if the vibrations can build up to a point where the structure can be damaged.

eg: unwanted resonances in loudspeaker enclosures, auditoriums, etc. that make some frequencies sound much louder than others

vibrations in badly-set car engines at certain "revs"

driving fast over an uneven road at a certain speed can cause the suspension in a vehicle to resonate and bounce wildly (especially if the dampers are faulty!)

vibrations induced in buildings or bridges by the wind, earthquakes, etc.

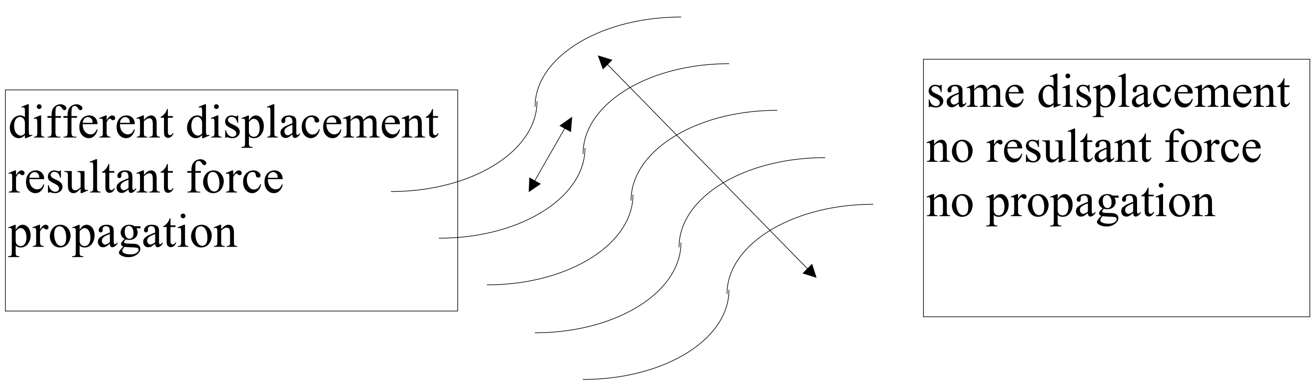
DIFFRACTION

So far we have considered interference between two (or more) separate waves.

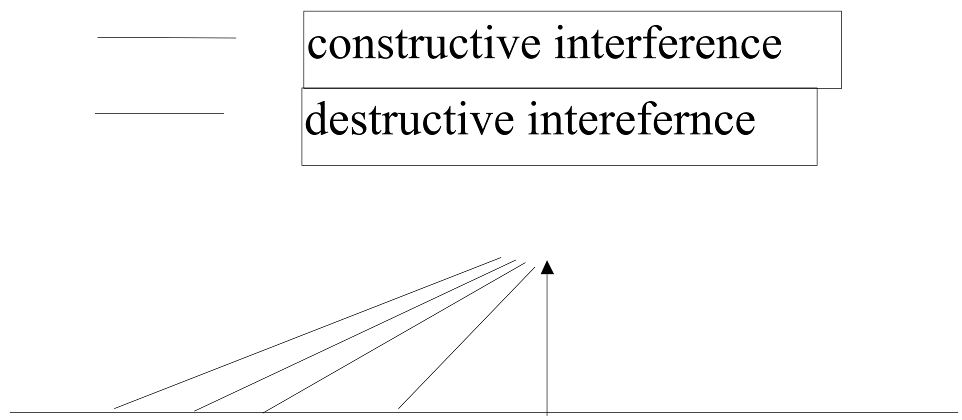
A wave can even be considered to interfere with itself all the time!

A disturbance in a 2-D or 3-D medium normally causes a wave to spread out in all directions (eg: dropping a pebble in a pond)

Yet a wave has a stable wavefront and propagates in a straight line. Physically, this can be explained because since adjacent particles in the wavefront have the same displacement, there is no disturbing force in the direction along the wavefront.



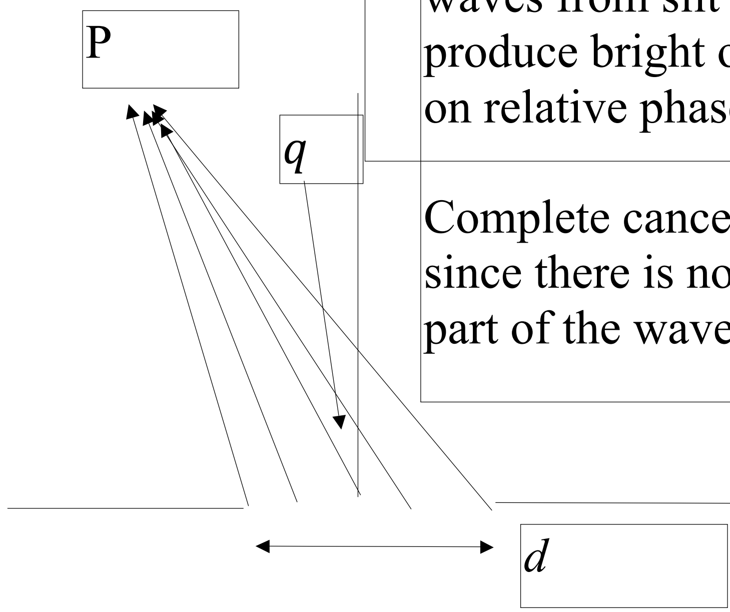
On a “wave” basis, this can be explained in terms of a wave “trying” to spread out in all directions, but interference cancels out all these attempts except the straight-on path.



In this example of a plane wavefront, if the wavefront is long enough (or infinite) for every constructive ray there will be a destructive one and so only the straight-through ray persists.

This condition, however, assumes an infinitely long wavefront. If we have an obstruction blocking part of the wave, this condition no longer holds at the edge of the obstruction.

A *very rough* idea as to what is happening is given below:



waves from slit interefere at P to produce bright or dark fringe depending on relative phase.

Complete cancellation is not automatic since there is no contribution from the part of the wavefront blocked off.

The resultant intensity at P can only be obtained by *integrating* the contributions of all the waves originating within the slit. However, it can be shown that:

- There is an intensity maximum opposite the centre of the slit

- There are intensity minima at angles to the straight-through beam given by $\sin\theta = n\lambda / d$ $(n \neq 0)$

- There are intensity maxima at angles to the straight-through beam given by $\sin\theta = (n + \frac{1}{2})\lambda / d$

Consequences:

- Appreciable diffraction only occurs for slit widths comparable to the wavelength. To diffract light, apertures about 0.1mm are needed. Sound diffracts easily through metre-sized apertures.

- Diffraction also occurs at edges and around shadows. In particular, there is a bright spot at the centre of the shadow of a small circular object (the Arago spot)!

- Intensity of fringe pattern of double-slit experiment is modulated by diffraction pattern from each slit

- The image formed by a lens or other aperture is in fact a diffraction pattern! We can only discern two closely-spaced objects if their diffraction patterns are distinct. This leads to the *Rayleigh criterion* for the resolution of an optical instrument, q » 1.22 l / d

- A plane wave has a definite direction of motion but its "position" perpendicular to this direction is undefined. If you try to restrict this position, the wave no longer has a definite direction.