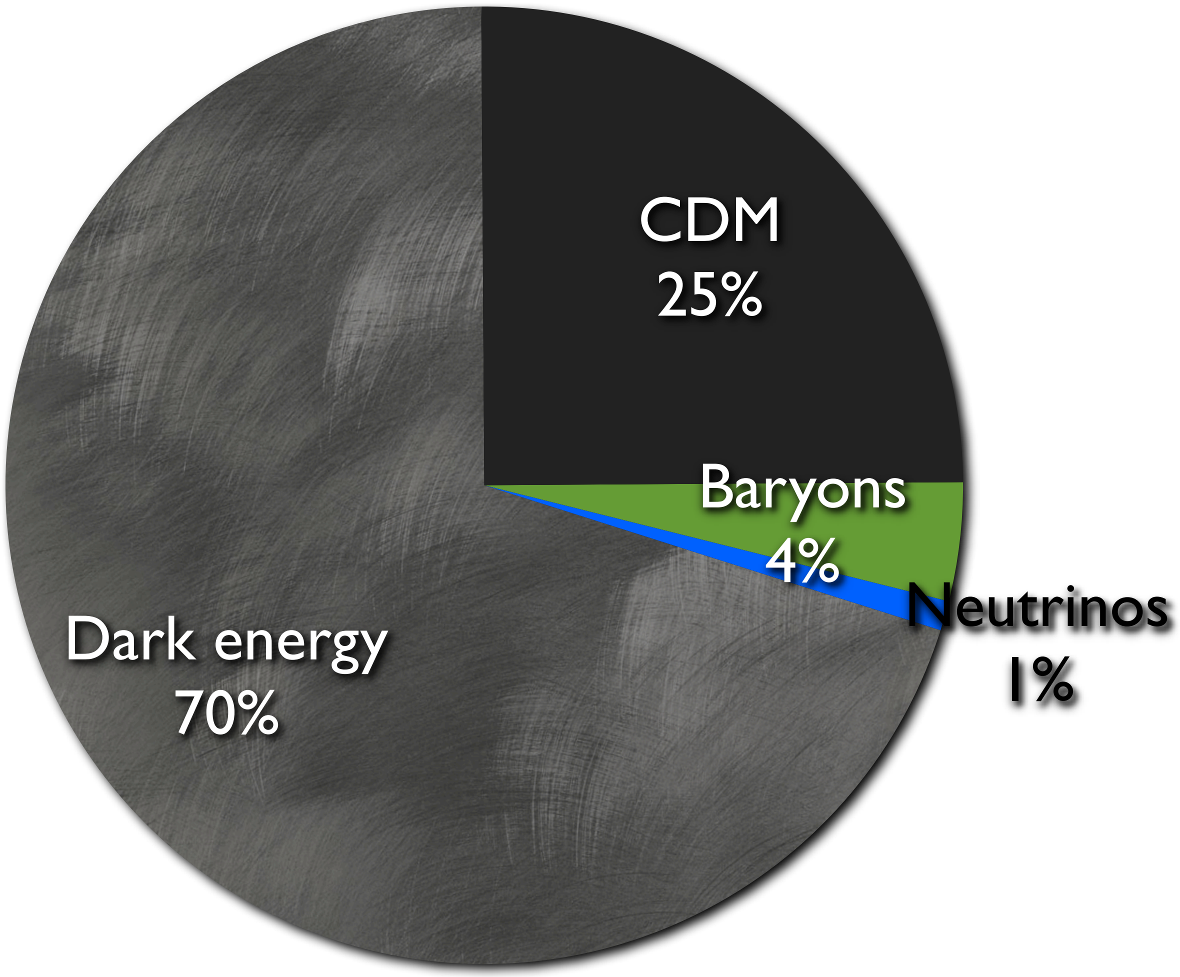
Revision: The cosmological model



Flat universe:

$$\Omega_{tot} = \Omega_m + \Omega_{DE} = 1$$

Initial conditions given by Inflation

Expansion of the Universe: parameterised by Hubble rate

H

H_0 = present value

Dark matter: non-luminous, no interaction with other components

Ω_{CDM}

Baryonic matter: can be observed via EM interactions

Ω_b $\quad \Omega_m = \Omega_b + \Omega_{CDM}$

Dark energy: has negative pressure, causes acceleration of expansion

Ω_{DE} $\quad w_0$ $\quad w_a$

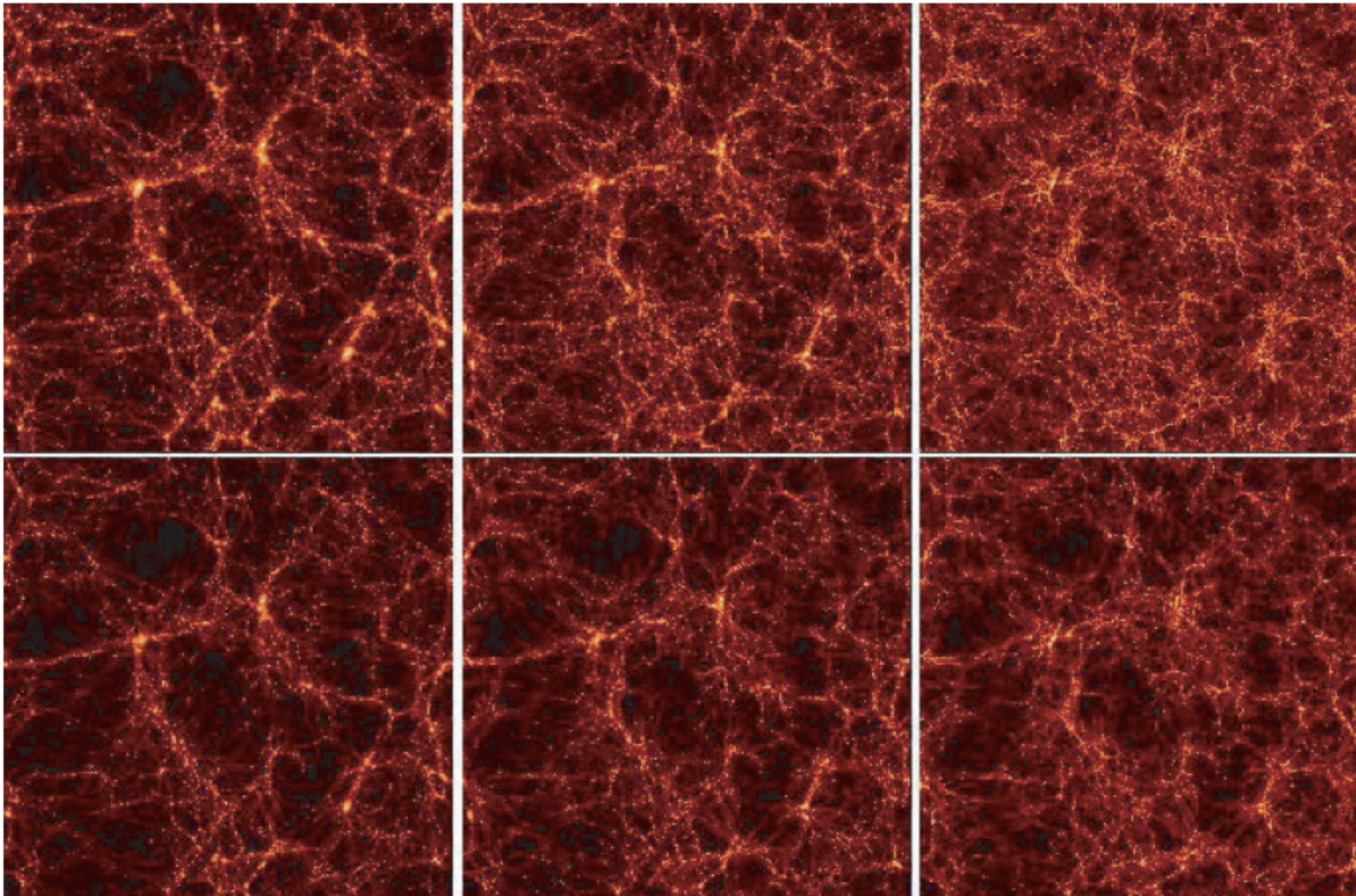
Initial conditions: a scale-invariant power spectrum of perturbations

n

Large scale structure is sensitive to cosmological parameters

Flat Universe with dark energy

Open Universe, no dark energy



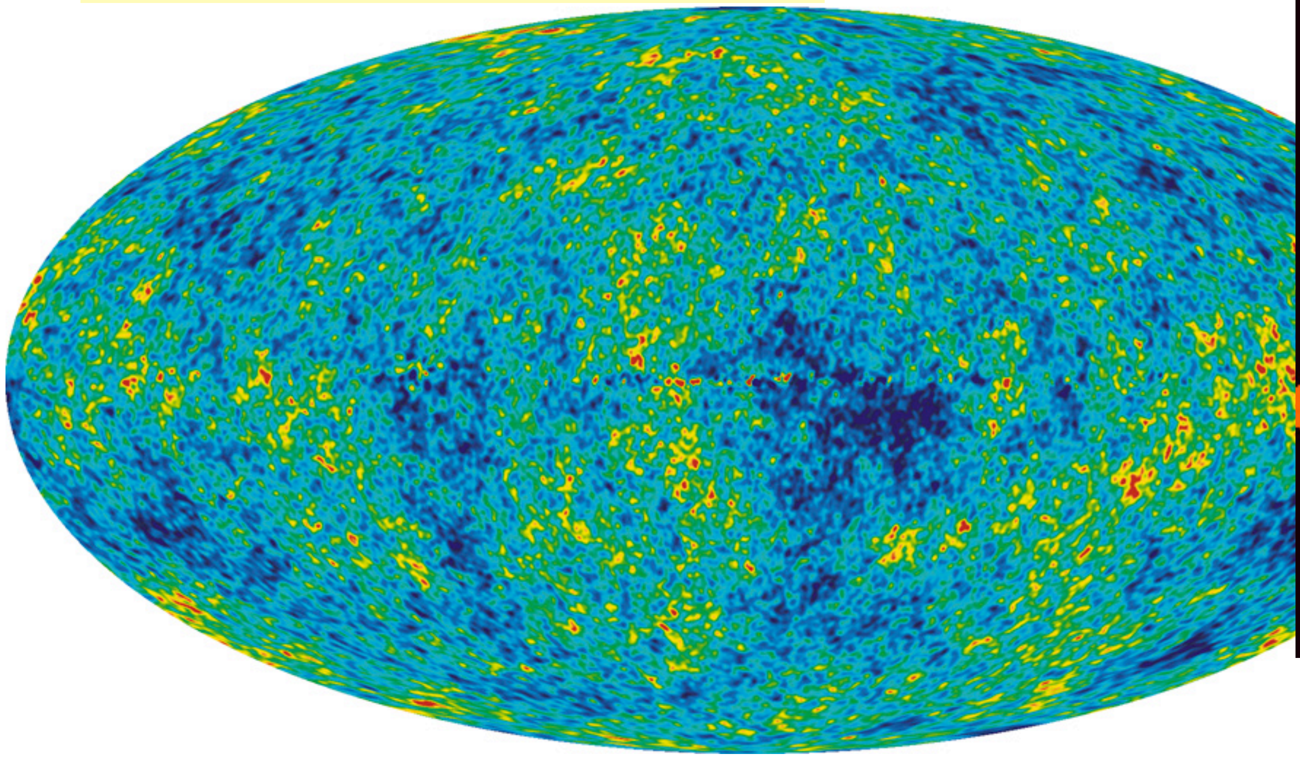
z=0 z=1 z=3

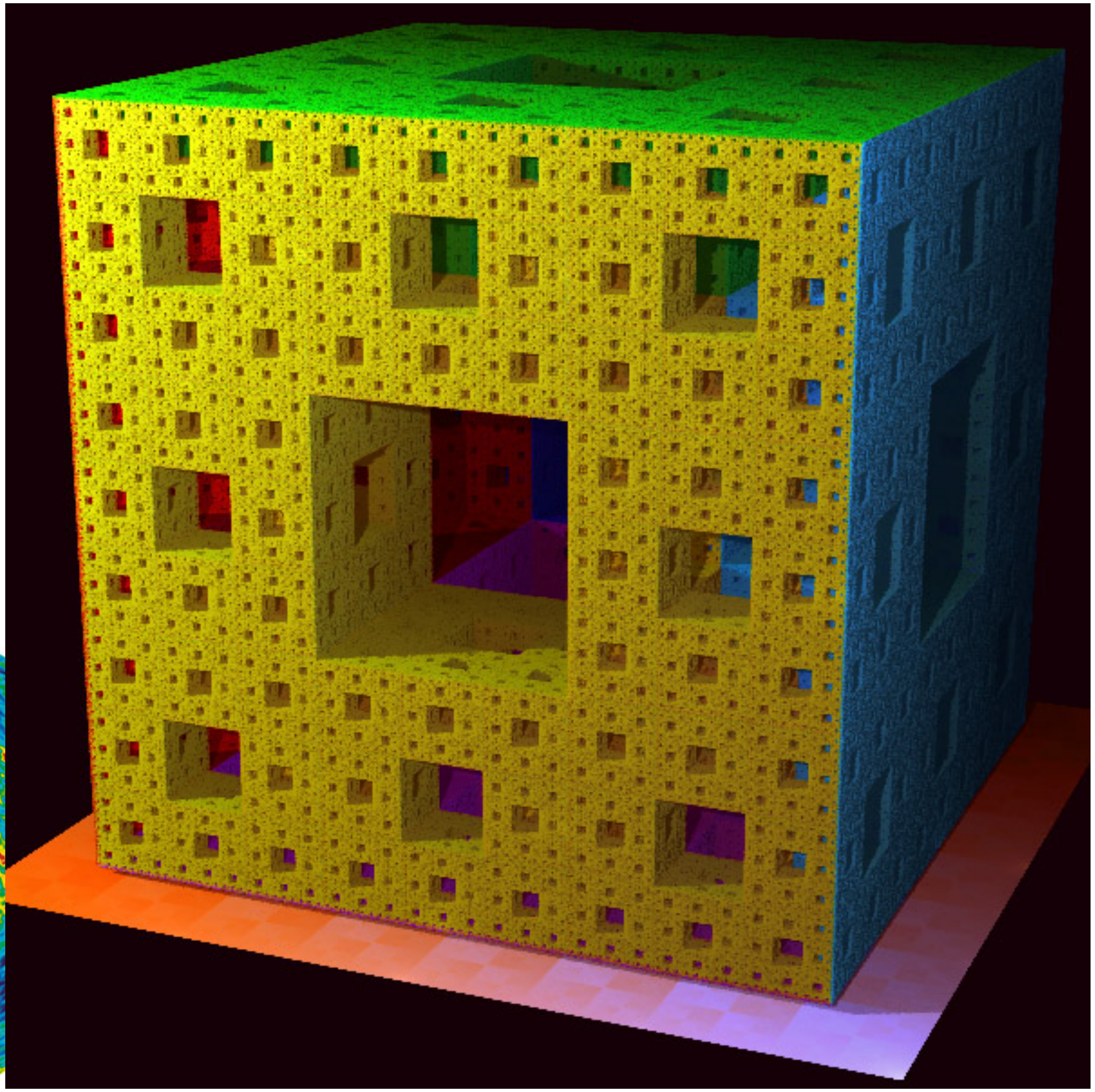
Initial conditions

CMB:
picture of perturbations in very early Universe.

Perturbations are fractal, so obtain a scale-invariant power spectrum.

Also called Harrison-Zel'dovich spectrum



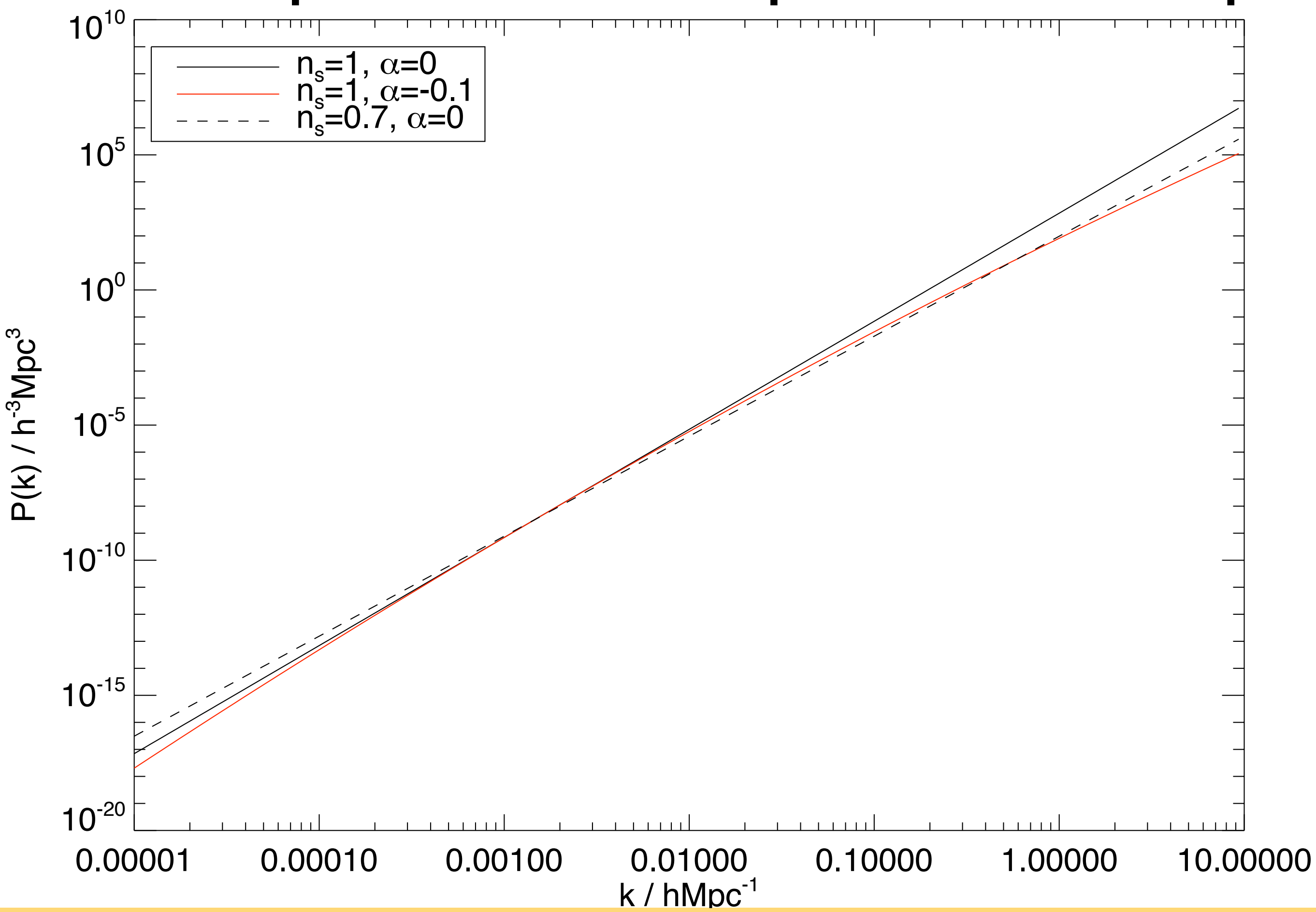


Uncertainty over initial conditions

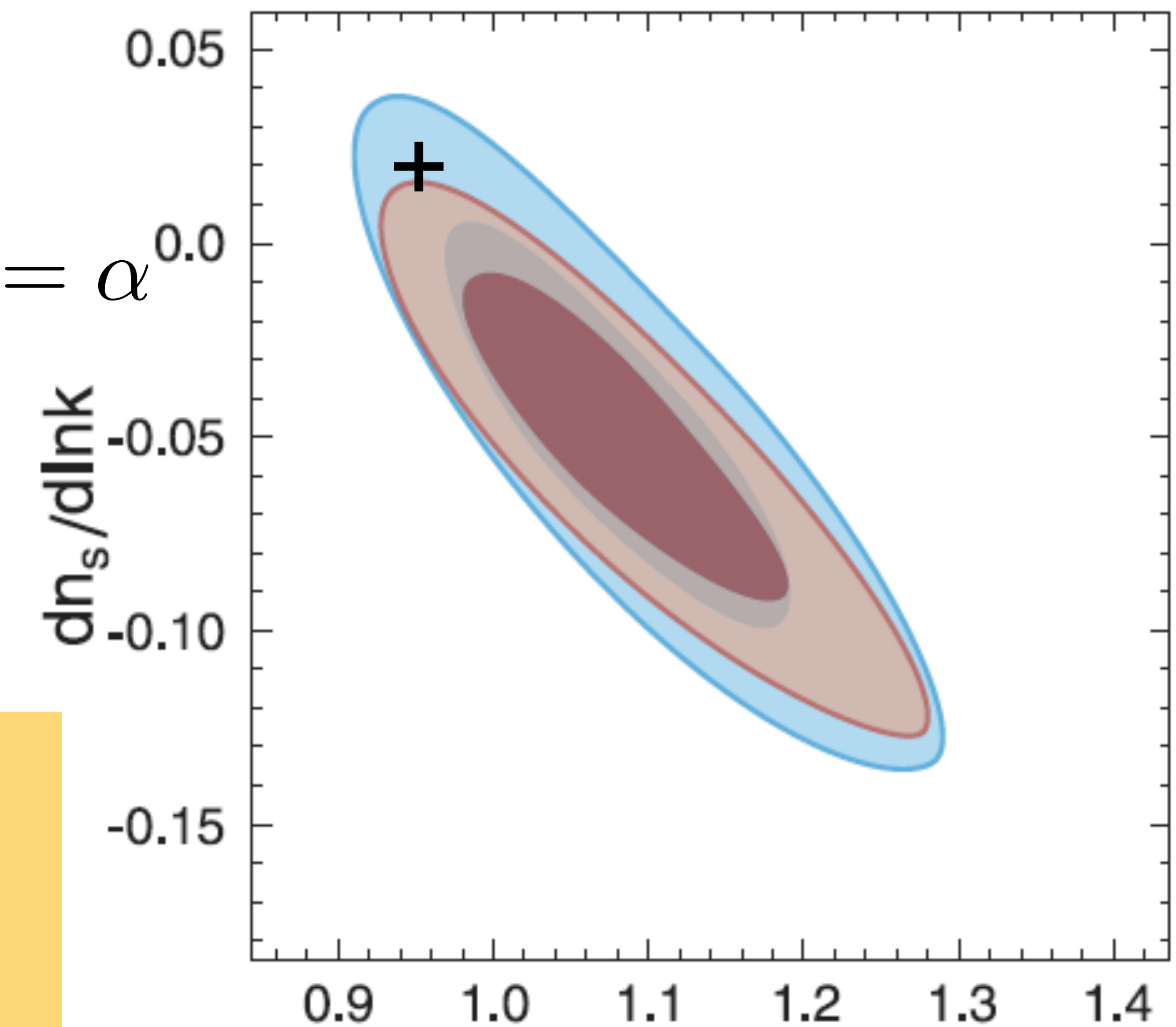
Power law (simplest model): primordial spectral index

Two parameters: amplitude and slope

$$P_{\mathrm{prim}}(k) = A_s k^{n_s}$$



$= \alpha$



- Motivated by Inflation
- Agrees with observations
- **BUT does the slope vary with scale?**
- Observations of CMB **not conclusive**
- Introduce parameter $\alpha = \mathrm{d}n/\mathrm{d}\ln k$
- Describes dependence of the slope n on scale k
- Therefore, we now have 3 parameters:

$$A_s, n_s, \alpha$$

$$n_s(k) = n_s(k_0) + \frac{1}{2}\frac{\mathrm{d}n_s}{\mathrm{d}\ln k}\bigg|_{k_0} \ln\left(\frac{k}{k_0}\right)$$

Neutrinos

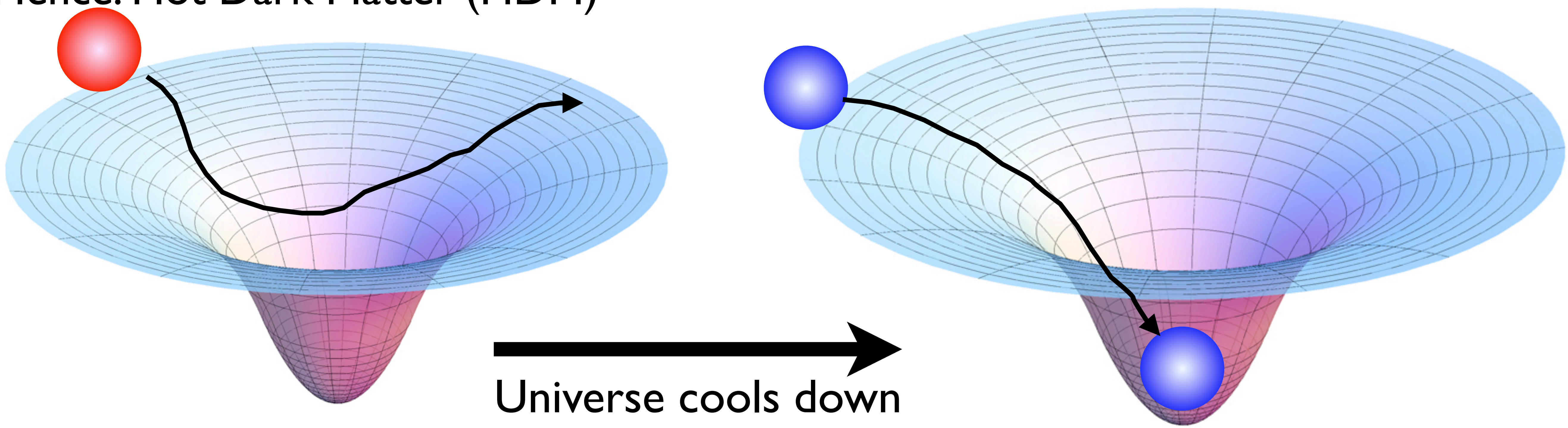
- Particle in Standard Model, observed experimentally
- Super-KamiokaNDE detector (1998): By observing energy differences, we conclude that **neutrino have nonzero mass**

- 3 neutrino species (Standard Model)
  Possibility: Some massless, some massive (?)

- Neutrinos contribute to total mass content of the Universe (like baryons and dark matter)
- But their small mass means that they have a particular effect.
  - They are massive: cannot treat like photons.
    They have very small mass: cannot treat like baryons.
- If different species have different mass, they will behave in different ways
- Cosmological effect therefore depends on the **total mass** and the **mass of individual neutrinos**
- **Neutrinos affect the distribution of matter on small scales.**
- **How?**

Effect of massive neutrinos on growth of structure

Relativistic neutrinos behave like photons. When they lose energy, they start behaving like baryons.
But: very low reaction cross-section. Difficult to detect.
Hence: Hot Dark Matter (HDM)



Highly energetic, relativistic neutrino.
Escapes potential well.

Neutrino loses energy, becomes non-relativistic.
Falls into potential well.

➡ Large-scale structure formation is sensitive to total neutrino mass m_ν
➡ and number of massive species N_ν
➡ **Cosmological constraints** on neutrino mass $m_\nu \lesssim 0.6\mathrm{eV}$

•Cosmological parameters for model with
Dynamical dark energy
Dark matter
Massive neutrinos
A scale-dependent primordial power spectrum

dark energy: $\boxed{w_0 \quad w_a \quad \Omega_{\rm DE}}$

matter: $\boxed{\Omega_m \quad \Omega_b}$

neutrinos: $\boxed{m_\nu \quad N_\nu}$

Hubble: $\boxed{h}$

primordial power spectrum: $\boxed{\sigma_8 \quad n \quad \alpha}$

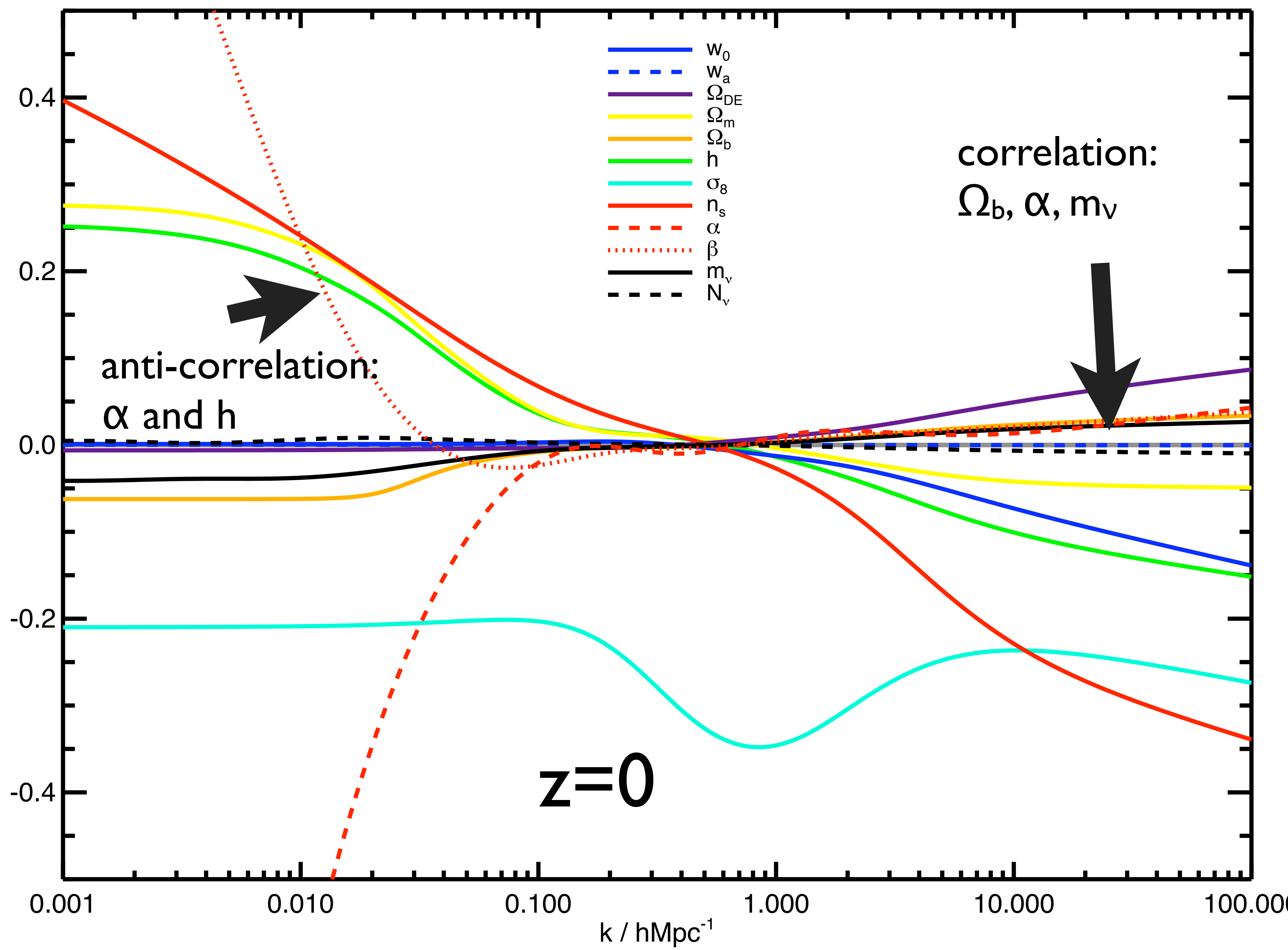
Simplest model has just 6 parameters, known in literature as ΛCDM

$$\Omega_\Lambda \quad \Omega_m \quad \Omega_b \quad h \quad \sigma_8 \quad n$$

Because we have many parameters, different values of different parameters can give us the same result.
Known as **parameter degeneracy**

Example of parameter degeneracy: effect on P(k)

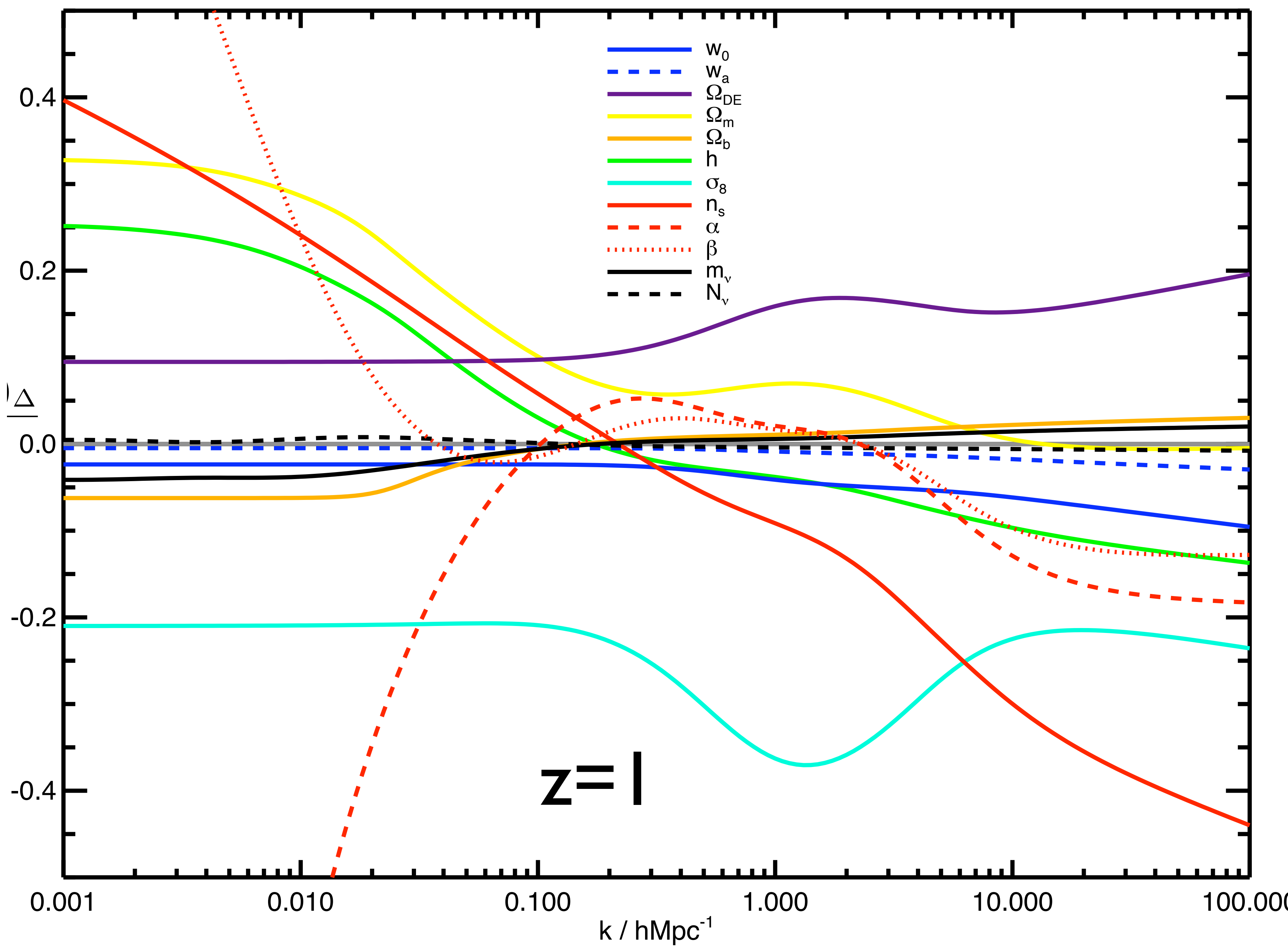
$$\text{Fractional change} = \frac{P(k)_{\rm fid} - P(k)}{P(k)_{\rm fid}}$$



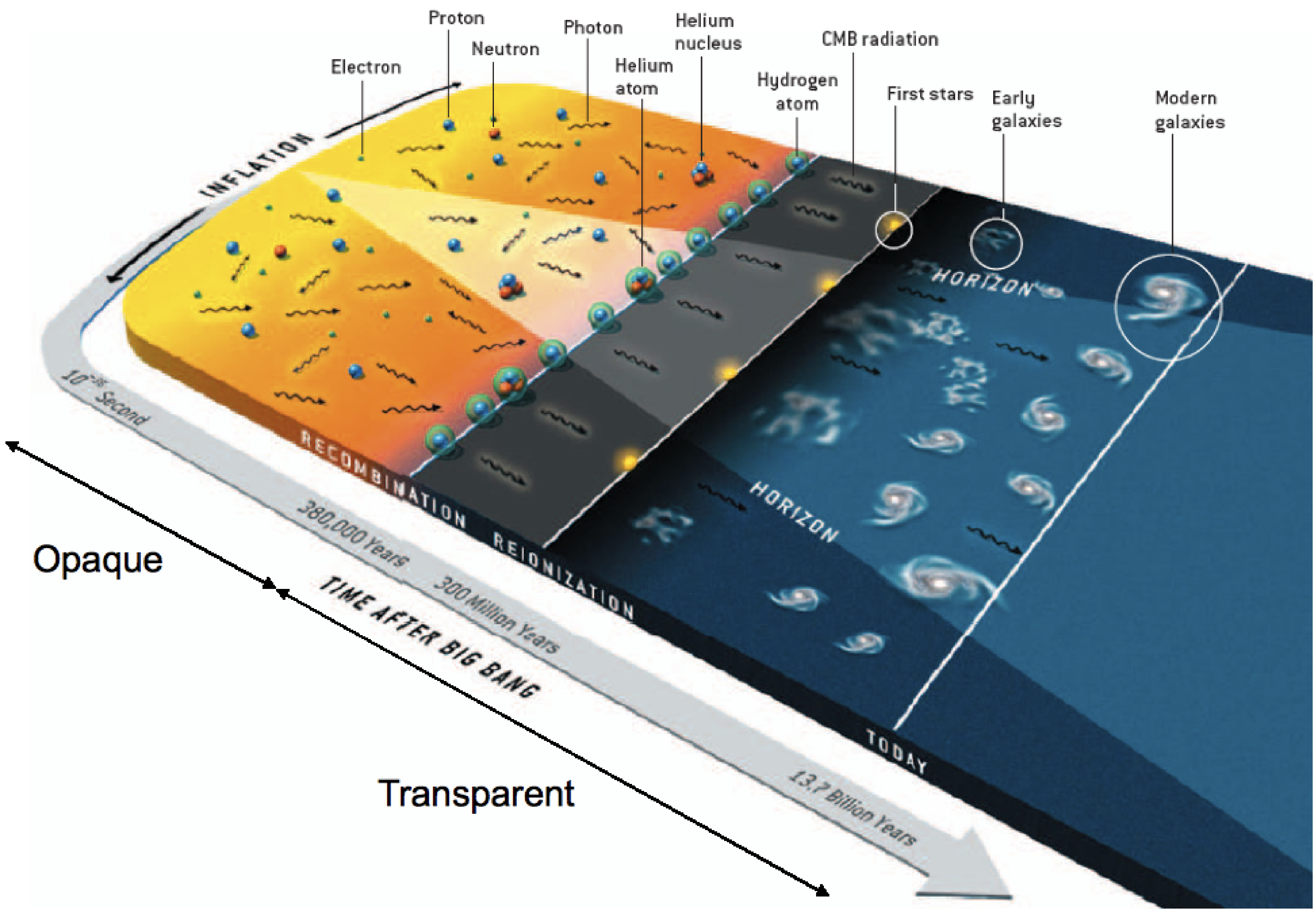
10% variation on each parameter

Effect on different scales and redshifts

$$\text{Fractional change} = \frac{P(k)_{\mathrm{fid}} - P_{\Delta}}{P(k)_{\mathrm{fid}}}$$



Evolution of the universe



Hu & White, Sci. Am., 290 44 (2004)

Cosmological probes

Standard candles: probe expansion

$z \sim 1$

- **Supernovae**

Map matter distribution: Growth of structure

- **Weak gravitational lensing**

Geometry

$z \sim 1100$

- **The Cosmic Microwave Background**

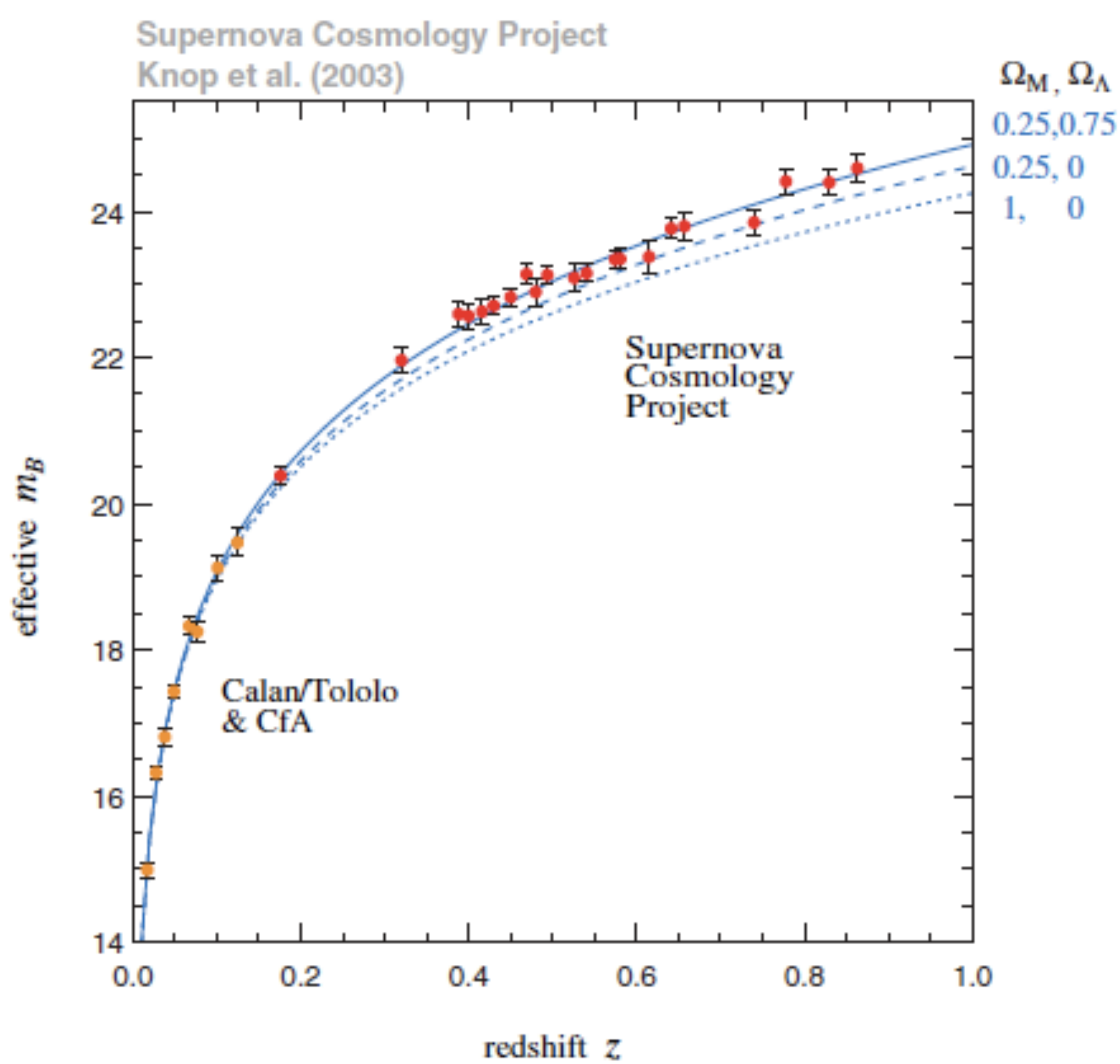
Initial conditions

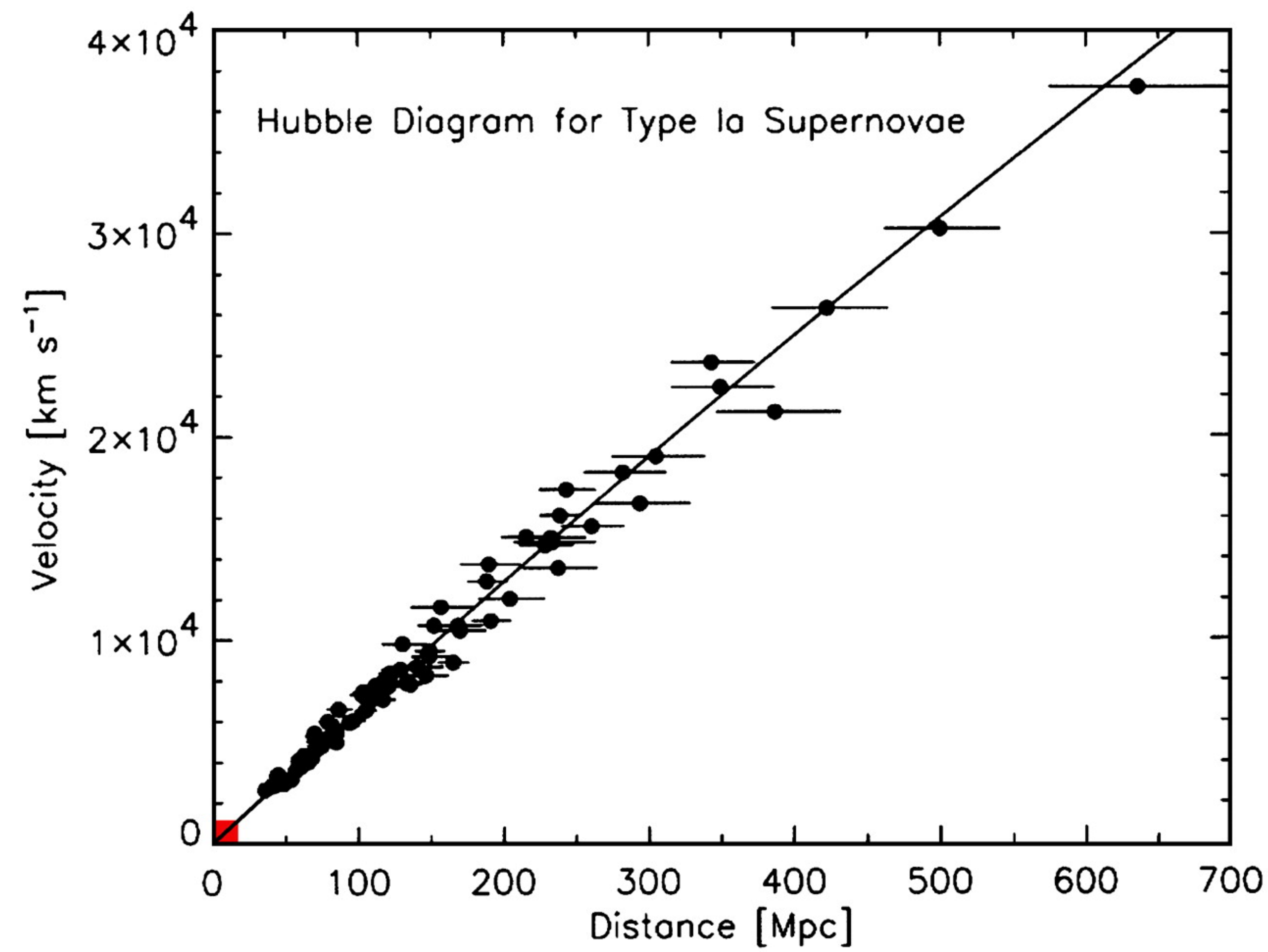
All of these probes can be used to constrain cosmological parameters. Some constrain certain sectors particularly well. Each has its difficulties.

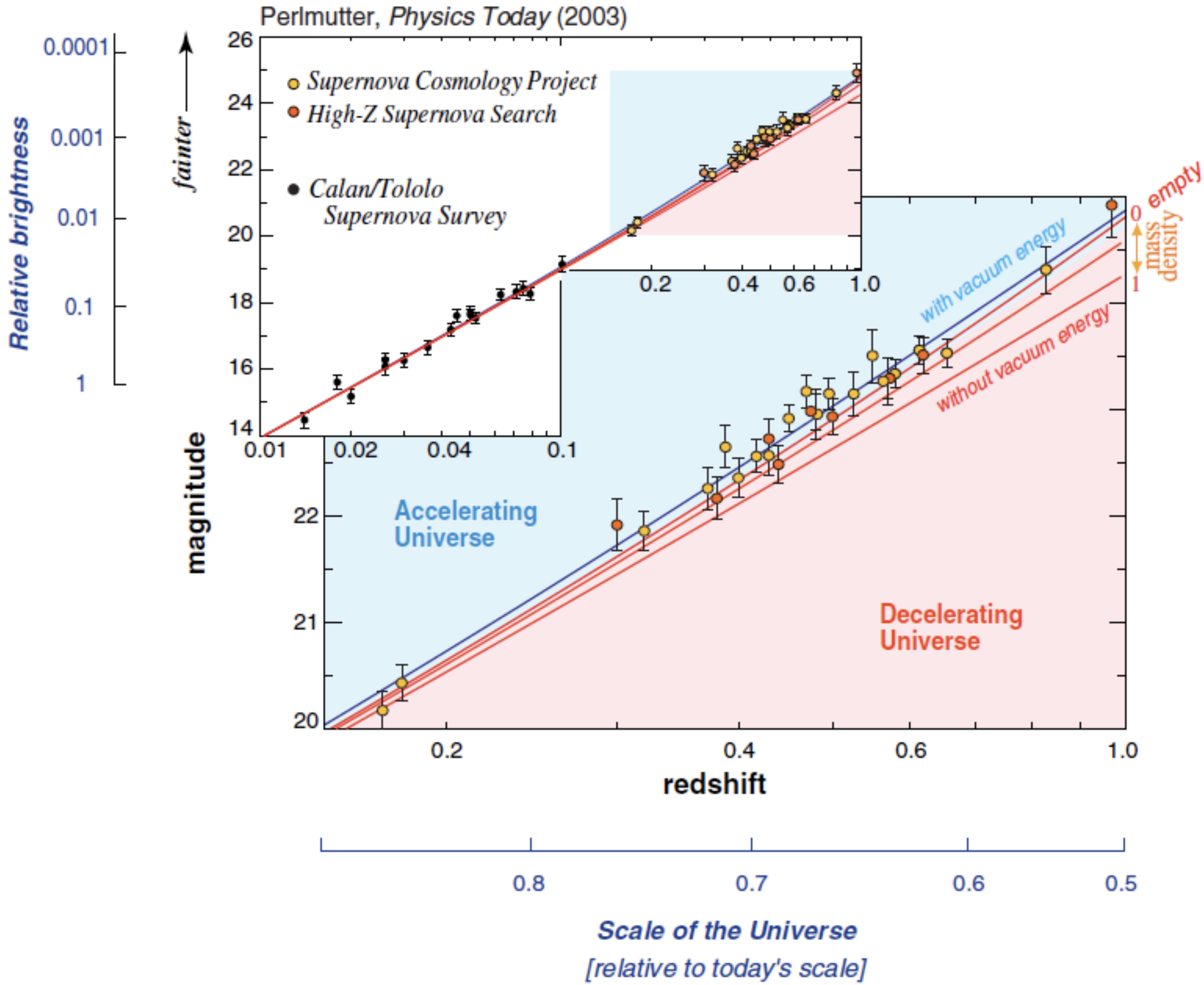
Cosmic variance: the statistical uncertainty inherent in observations of the universe at extreme distances. It is only possible to observe part of the Universe at one particular time, so it is difficult to make statistical statements on the scale of the entire universe.

Supernovae

Supernovae are **standard candles:** absolute magnitude is well-known
Use magnitude-distance relation to infer distance
Measure the velocity
Therefore measure the Hubble rate (at low redshift)

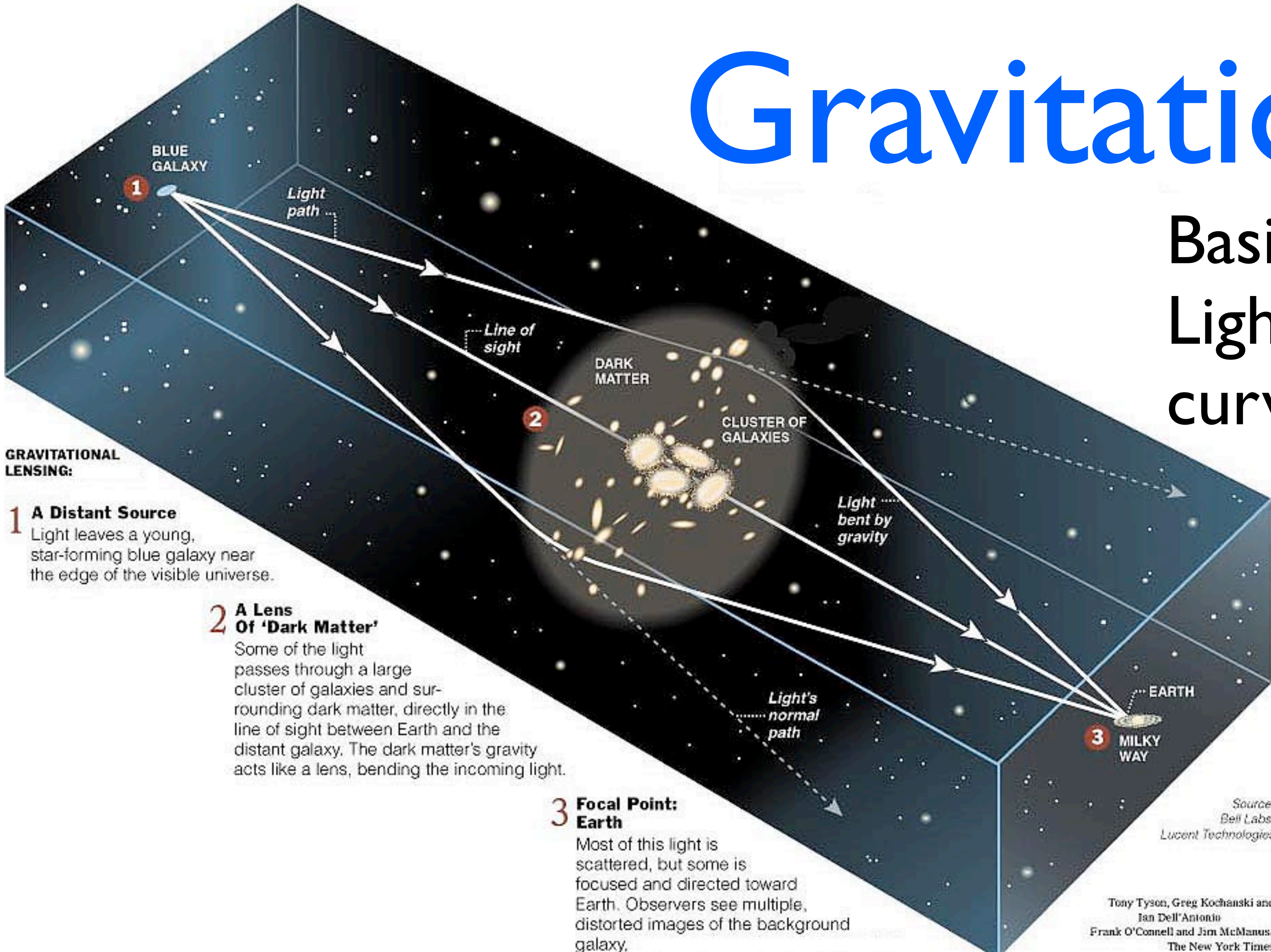






Difficulties: Intrinsic and observed brightness of supernovae depends on complex physics. Difficult to calibrate.

Gravitational lensing



Basic principle:

Light rays are deviated by curved spacetime.

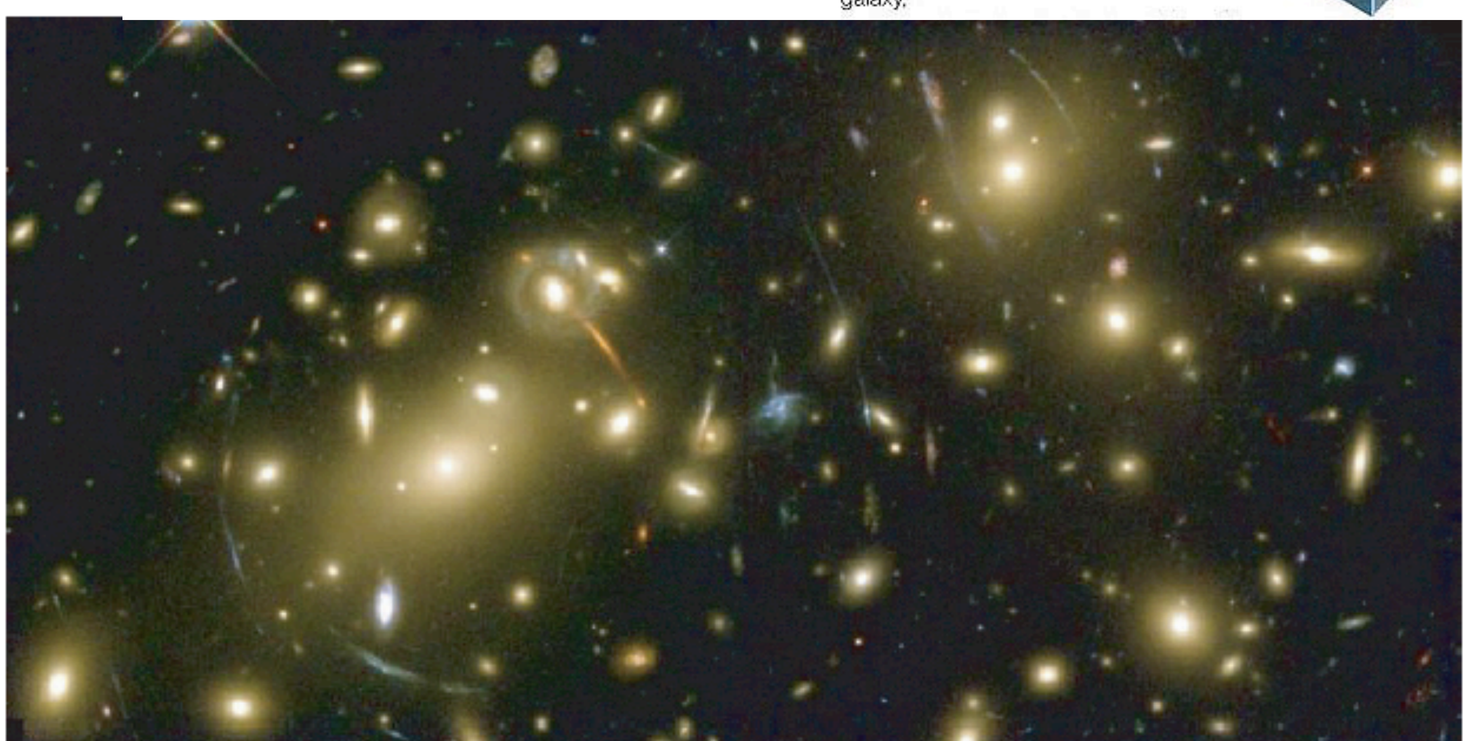


FIG 3.5.— Image of the rich galaxy cluster Abell 2218 showing many arcs and arclets that are images of background galaxies distorted by the gravitational tidal field of the cluster mass. (W. Couch, R. Ellis, NASA)

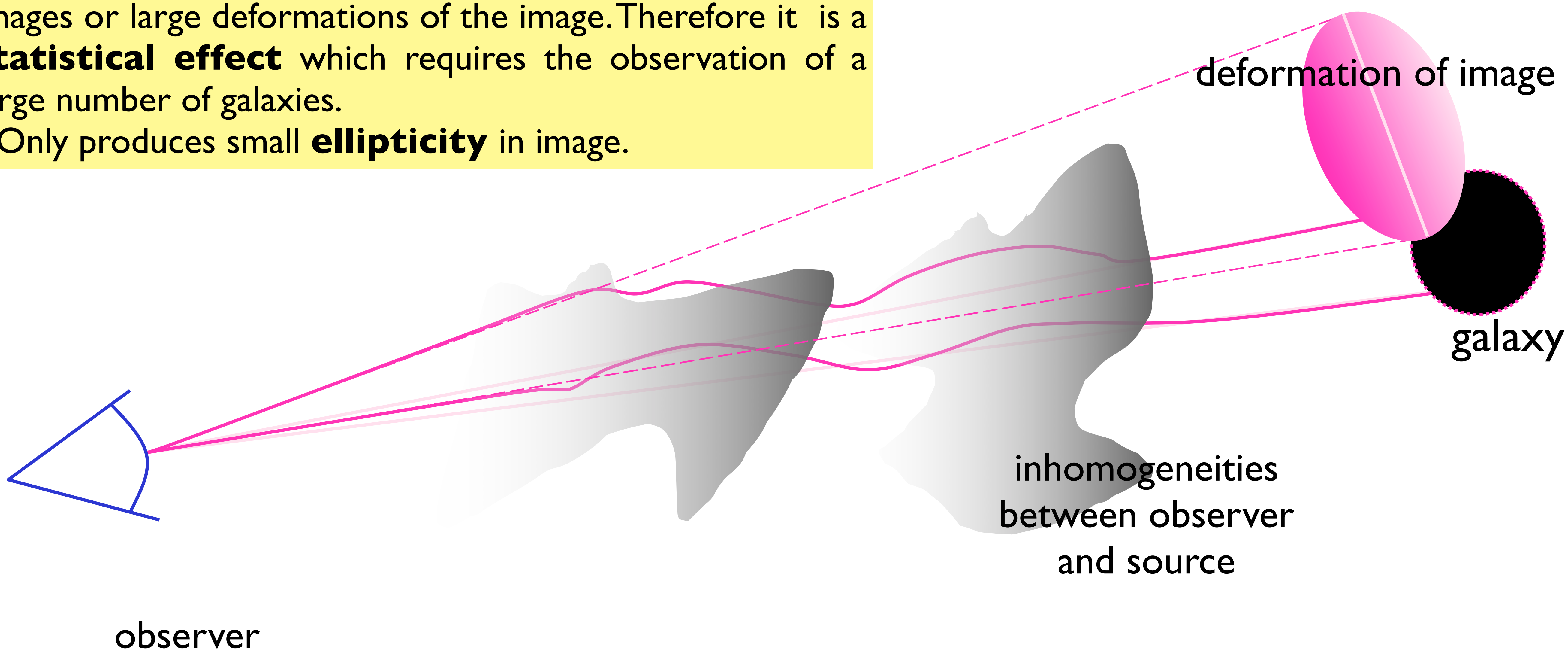
Strong lensing.

Occurs when observer, lens and source are aligned.

Arcs or multiple images are formed.

Weak gravitational lensing

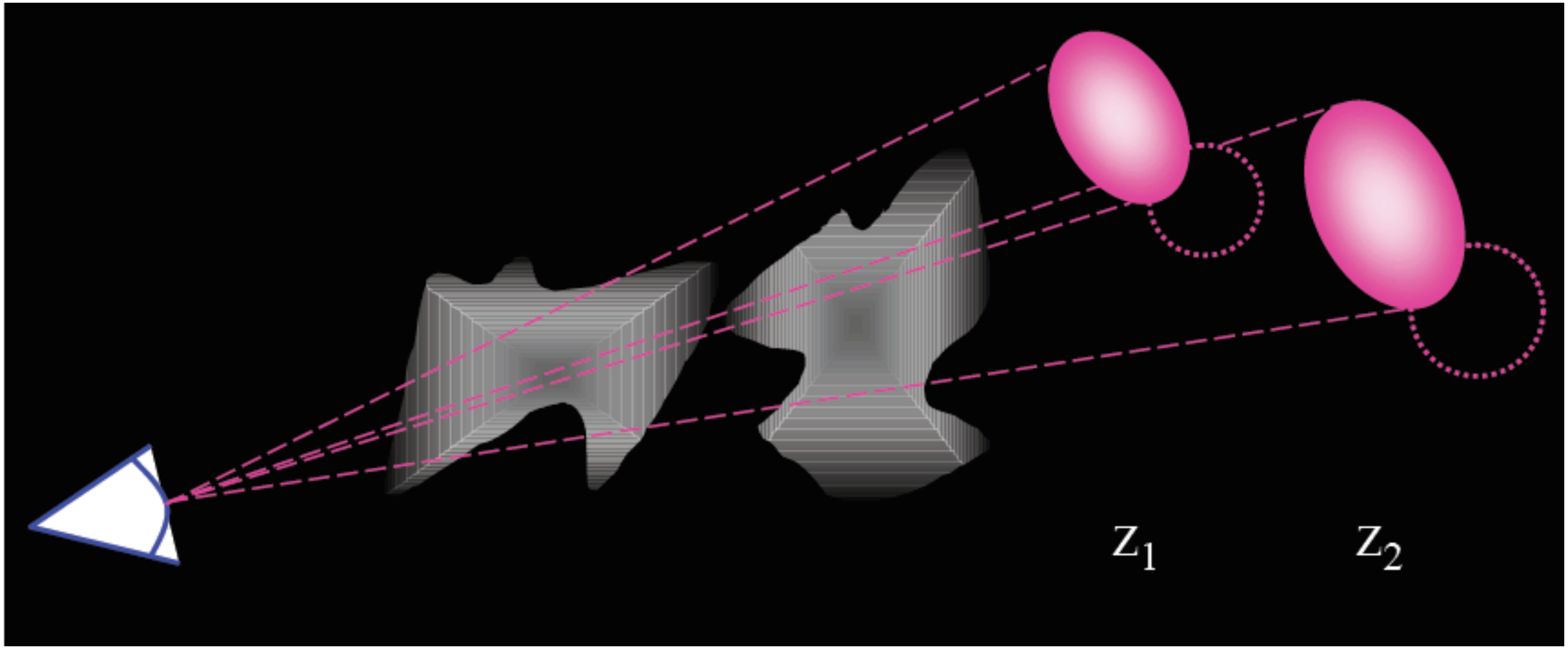
- Occurs when observer, lens and source are **not perfectly aligned**.
- Unlike strong lensing, this does not produce multiple images or large deformations of the image. Therefore it is a **statistical effect** which requires the observation of a large number of galaxies.
- Only produces small **ellipticity** in image.



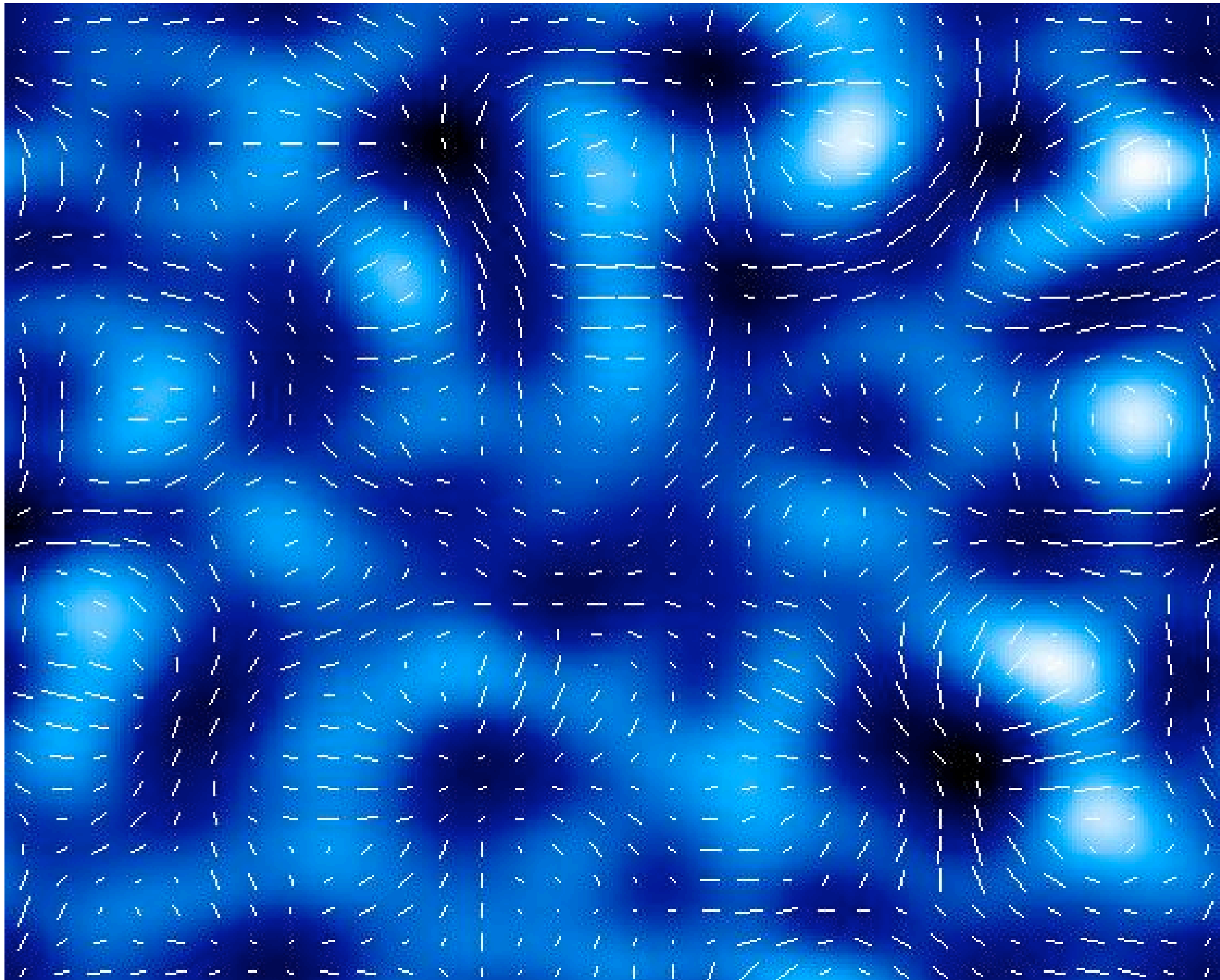
theory

Direct measure of the **distribution of mass** in the Universe without hypotheses about the properties of matter

Weak lensing tomography



- Put galaxies in redshift bins
- Measure redshift dependence of weak lensing signal
- Constrain redshift-dependent quantities: H(z), w(z) ...
- This technique probes both the **geometry** of the Universe and the **growth of structure**

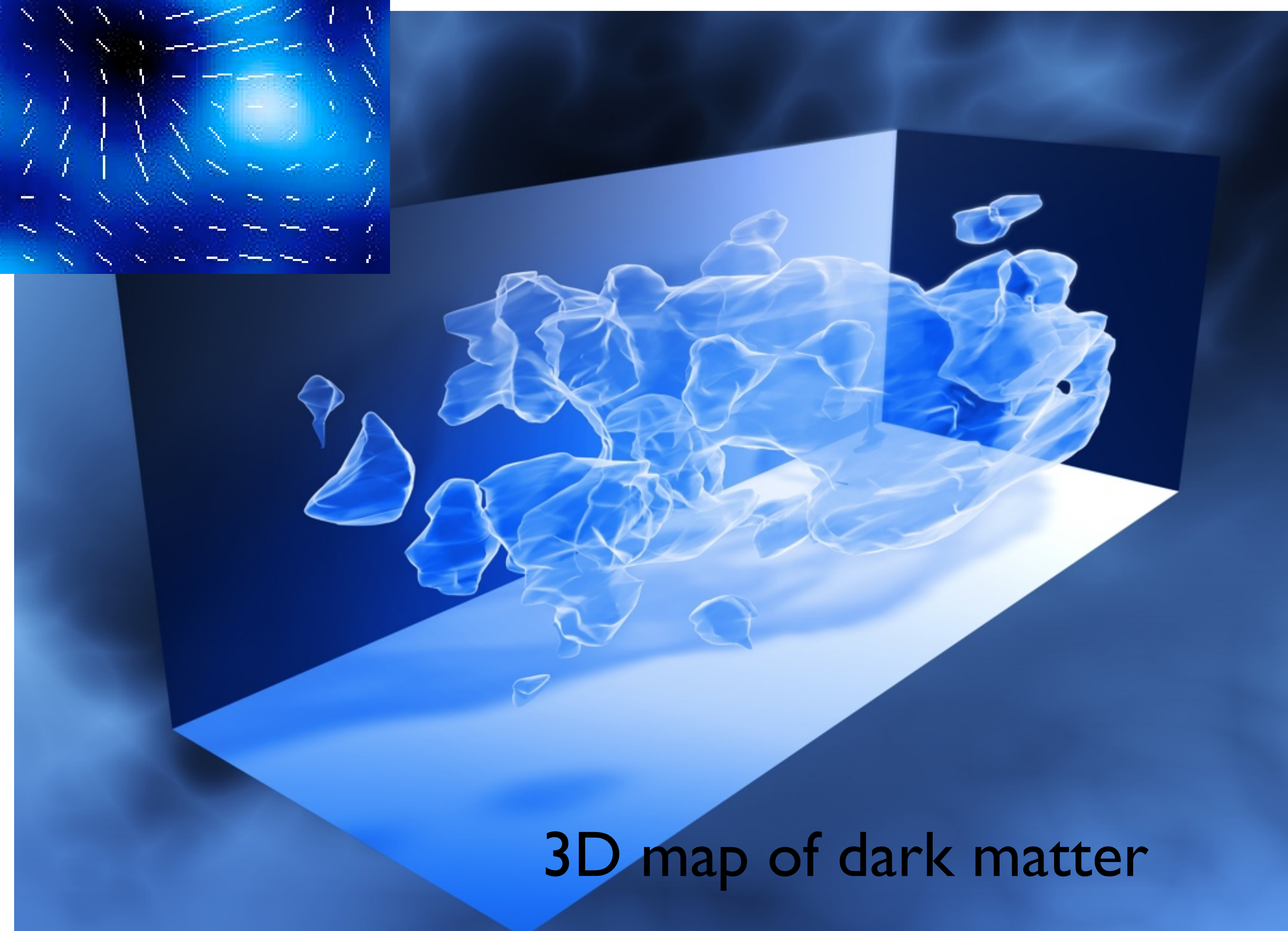


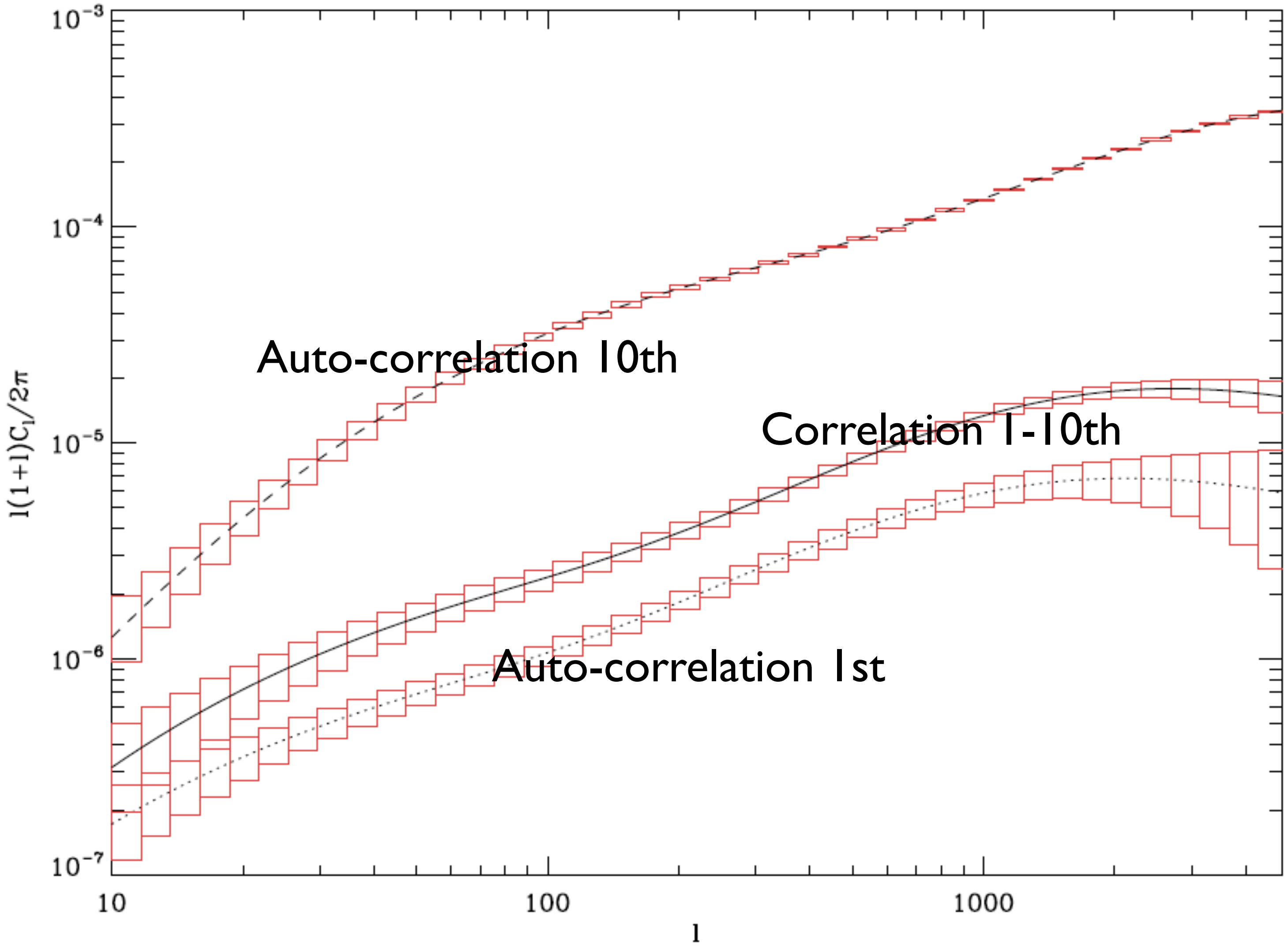
Map of mass distribution in the Universe

- allows us to **map dark matter**

Mass distribution given by **matter power spectrum**...
... which depends on cosmological model

Shear map: shows ellipticity correlation (or shear) caused by the presence of matter inhomogeneties





Shear power spectrum:
shows correlation between ellipticity in different redshift bins.

It is a function of the matter power spectrum and the properties of the lensing survey.

Difficulties:

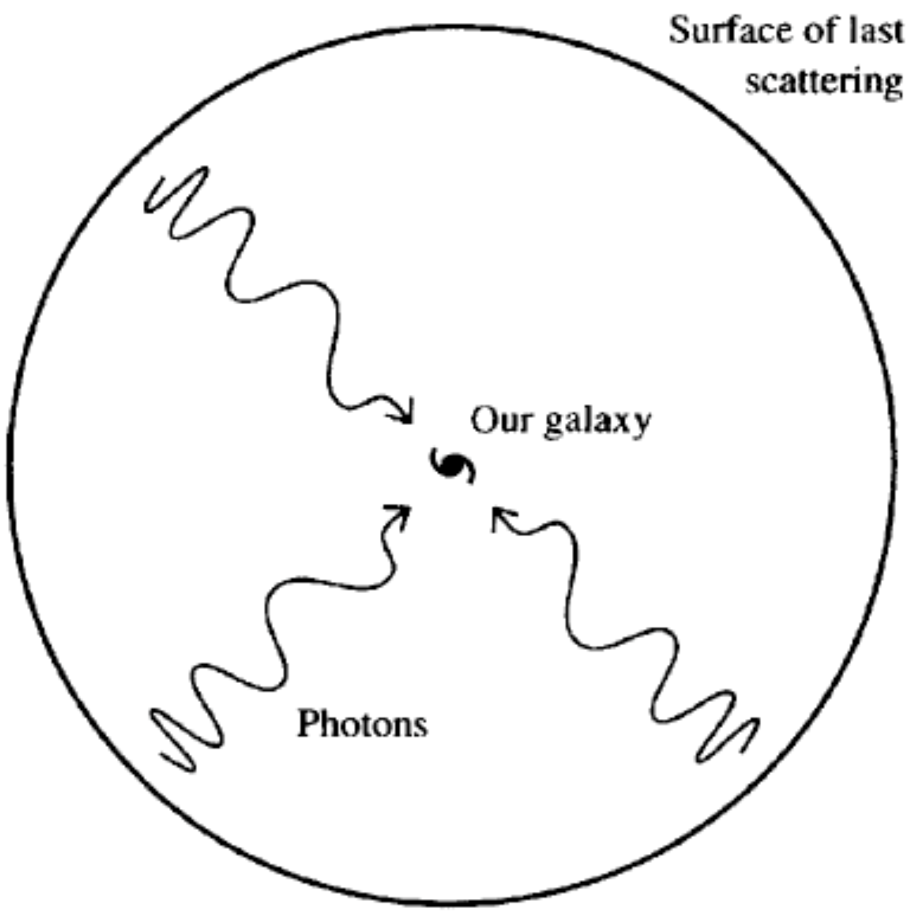
1) Galaxies have an **intrinsic ellipticity**. We need to measure the true lensing-induced ellipticity. Try to minimise by observing large number of galaxies.

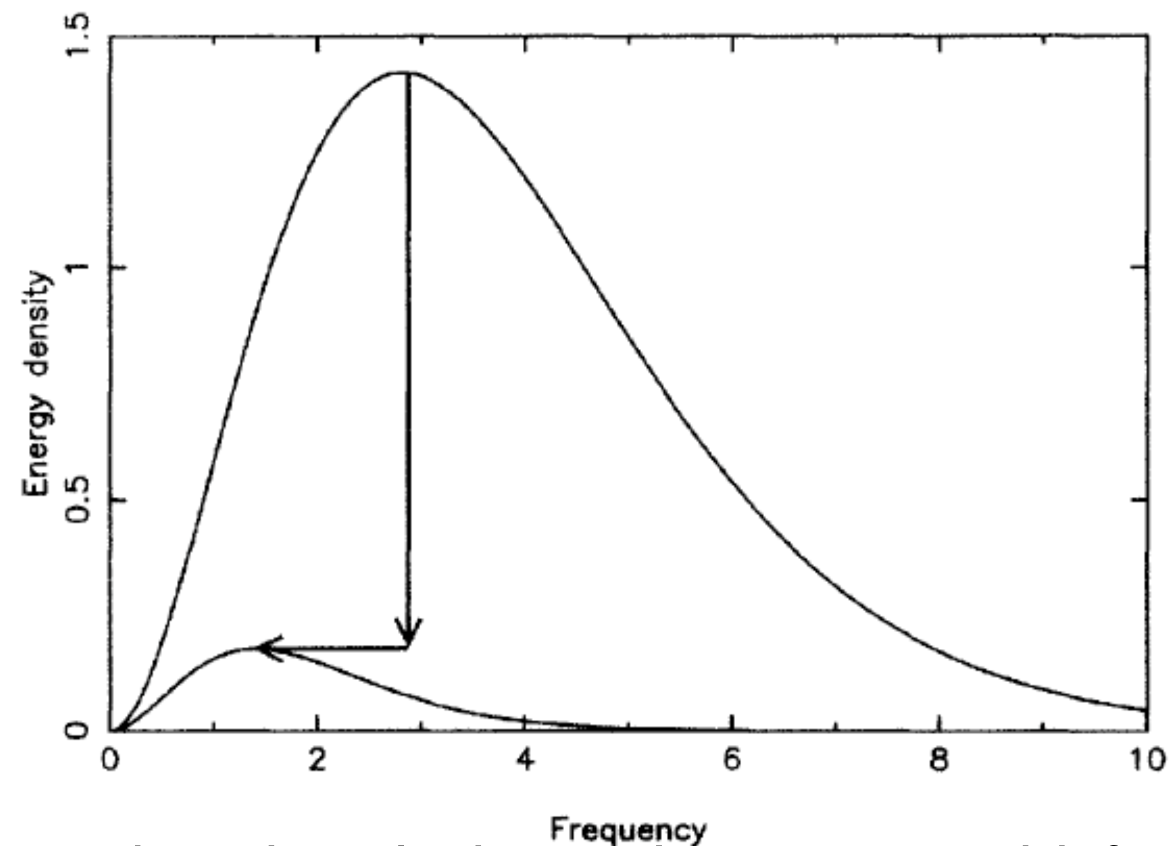
2) The presence of nearby galaxies can induce an alignment of galaxies (**intrinsic correlations**). We try to minimise this effect by observing over large redshift range.

3) **Measurement systematics:** All imaging systems introduce a small deformation of image (point spread function) which must be corrected.

4) **Theoretical uncertainties:** We need a very accurate model of structure formation to extract cosmological parameters from lensing. Small-scale effects are hard to model.

Cosmic Microwave Background





As universe expands and cools down, photons are redshifted and spectrum changes amplitude and position of peak

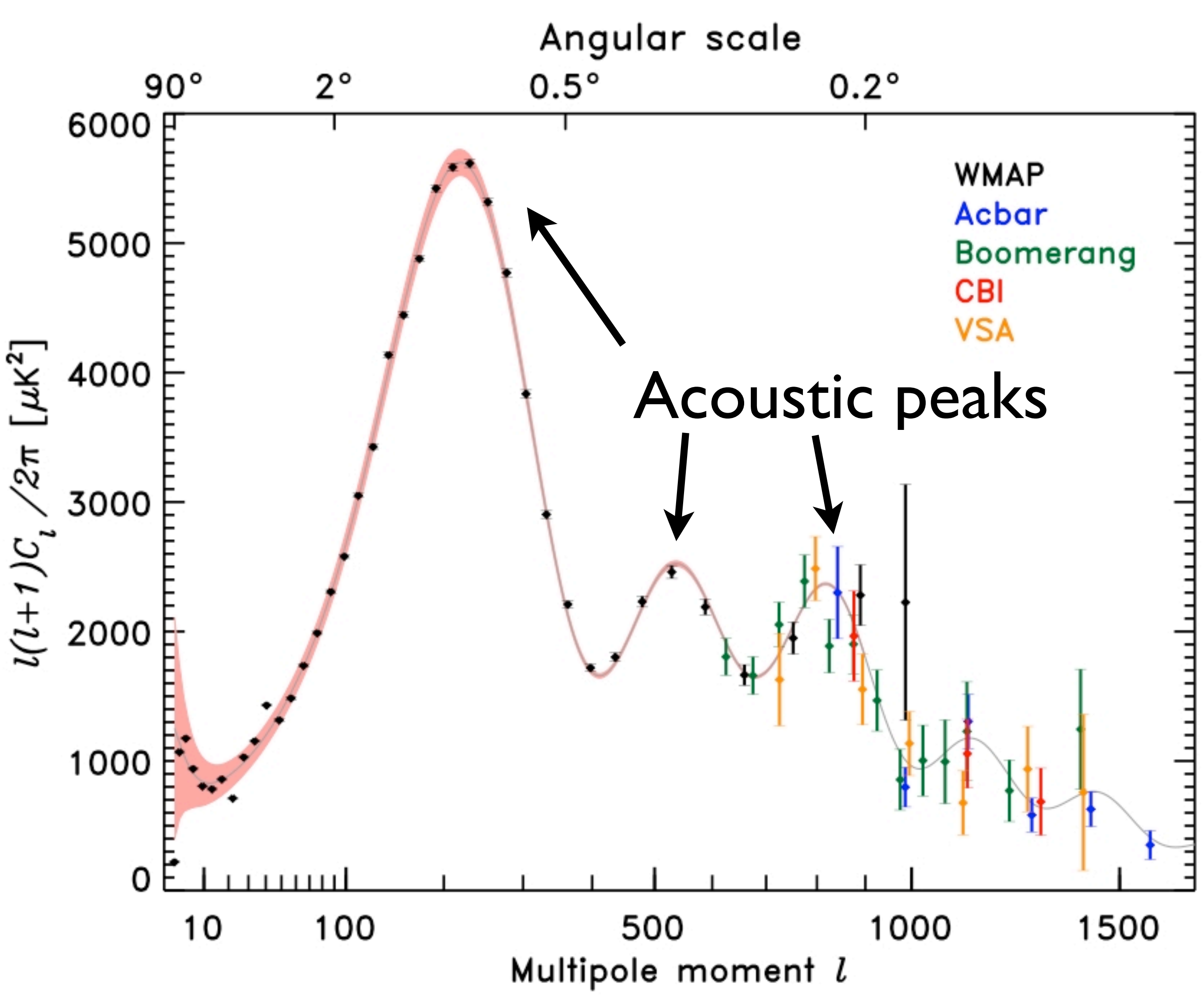
- CMB: Photons from **surface of last scattering**, when photons stopped interacting with matter.
- So the CMB contains an imprint of matter distribution at decoupling (universe only 380 000 yrs old)
- Provides very good constraints on **early universe and initial conditions**.
- Currently no evidence for any deviations from standard near scale-invariant primordial spectrum, but these are not ruled out either.
- CMB probes observe the temperature at very small angular resolutions, so can detect any fluctuations across small patches of sky.

Cosmic Microwave Background anisotropies

- CMB anisotropies: very small variations in temperature.
- Principally caused by two effects:

  Acoustic oscillations: Pressure of the photons tend to erase anisotropies.

  Diffusion damping: The gravitational attraction of the baryons makes them tend to collapse and form overdensities.
- These two effects combine to create **acoustic peaks**.
- Roughly correspond to resonances in which the photons decouple when a particular mode (i.e. 'size' of density perturbation) is at its peak amplitude.
- We can therefore deduce the properties of the initial perturbations.



Additional information from CMB:

- Polarisation - evidence of tensor perturbations (gravitational waves)
- Lensing of the CMB signal - further probe of recent structure formation
- Sachs-Wolfe effect - redshifting of photons due to expansion of Universe. Probe of structure formation in very early universe.

Combining constraints from several experiments

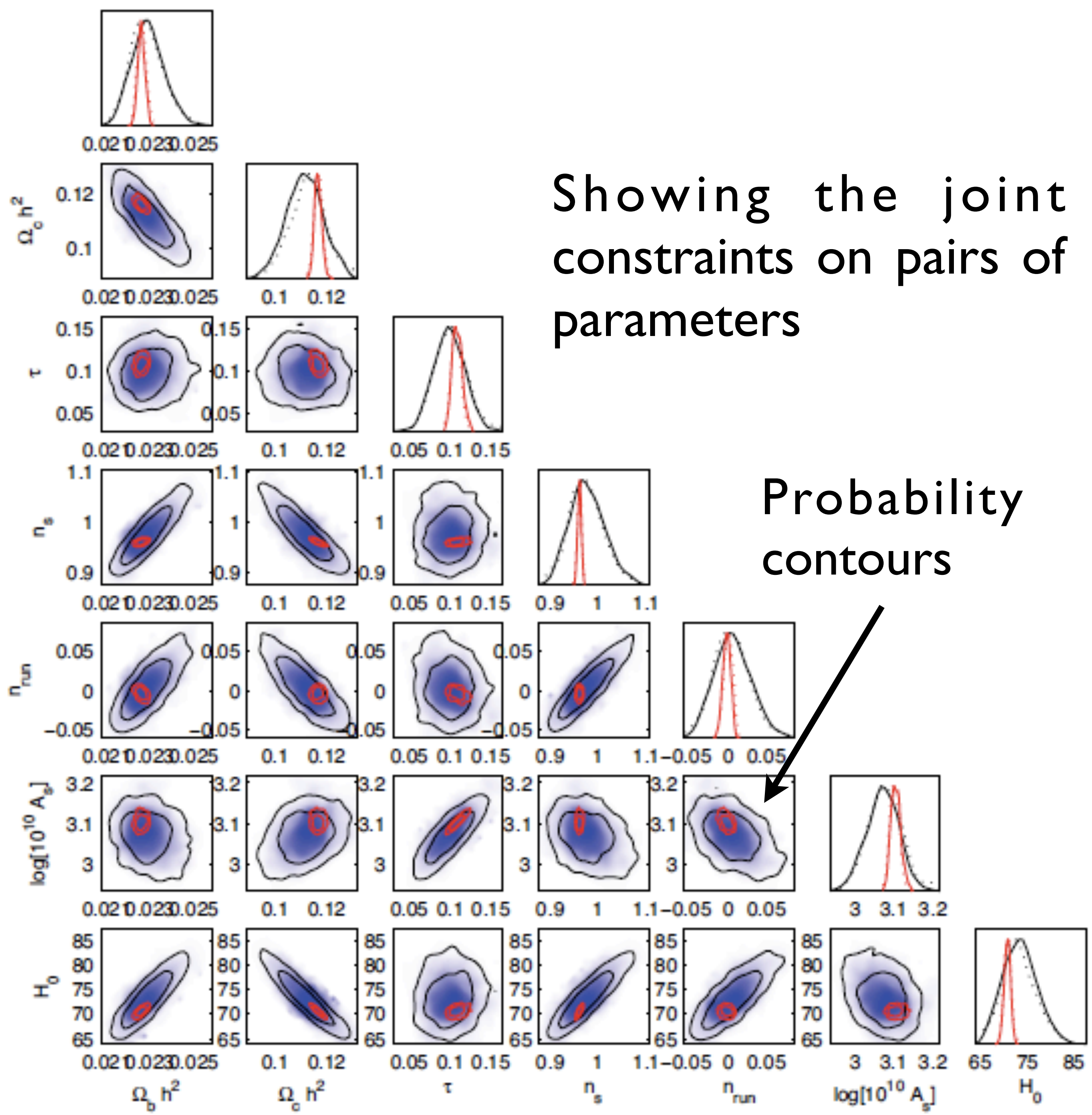
Adding information from two or more probes allows us to break parameter degeneracy in some cosmological sectors.

Lensing only **Lensing + CMB**

Parameter	Parameter precision (Lensing only)	Parameter precision (Lensing + CMB)
w_0	0.05633	0.04942
w_a	0.19297	0.17943
Ω_{DE}	0.05214	0.00644
Ω_m	0.00731	0.00389
Ω_b	0.02411	0.00091
m_ν/eV		
N_ν		
h	0.11337	0.00599
σ_8	0.01184	0.00461
n_s	0.02904	0.00332
α		
β		
FOM	130.99	357.12

DE

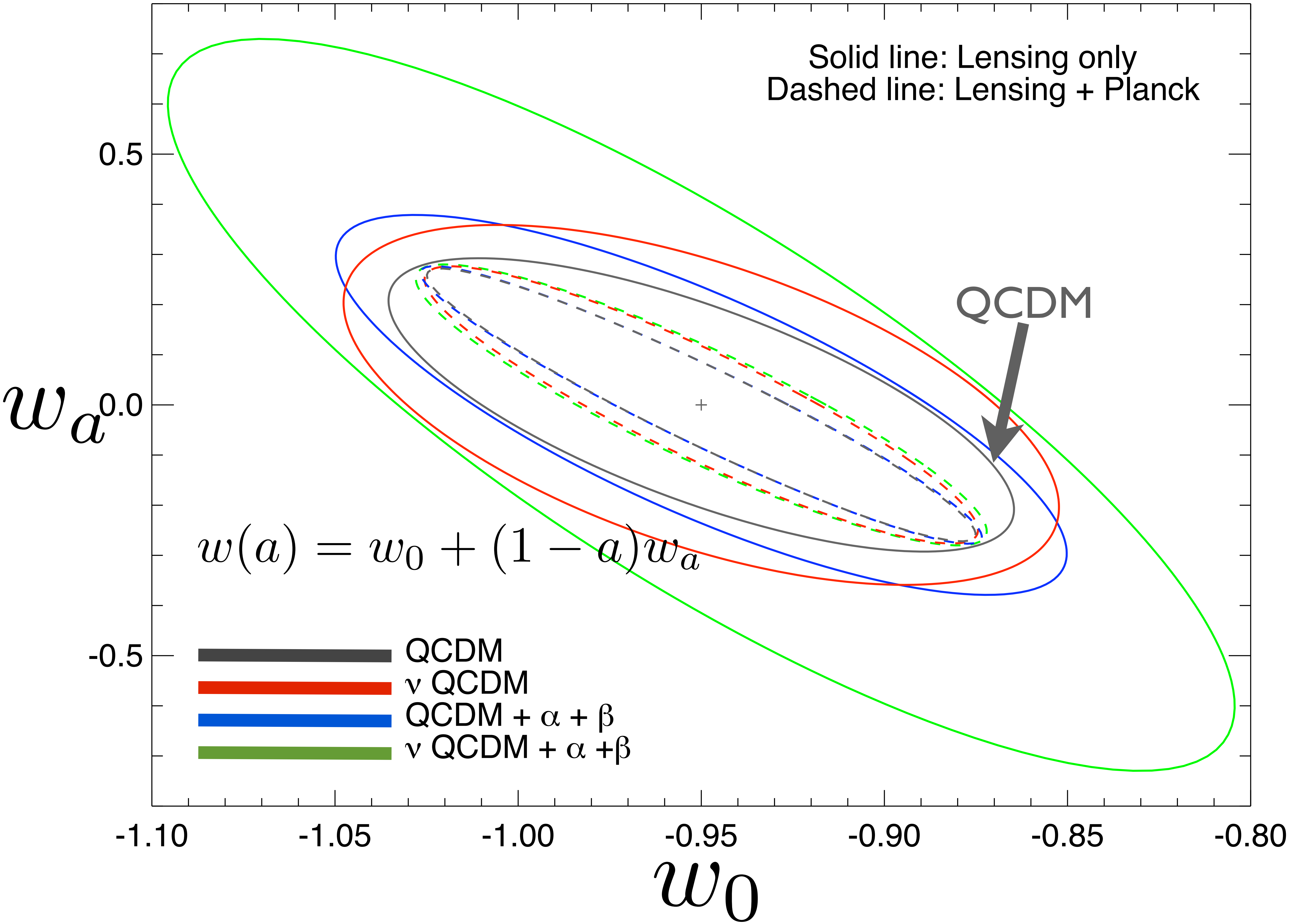
prim. sp. index



Showing the joint constraints on pairs of parameters

Probability contours

An example of combined constraints from future probes: the dark energy equation of state



Alternative explanation of acceleration?
Backreaction

- The FRW equations do not describe the expansion of an inhomogeneous space
- **The average behaviour of an inhomogeneous spacetime is not the same as the behaviour of the corresponding smooth spacetime.**
- Applying the field equations does not commute with averaging:

$$\langle G_{ab}(g_{ab})\rangle \neq G_{ab}(\langle g_{ab}\rangle)$$

➡ **average quantities do not satisfy the Einstein equation.**

- This is **the fitting problem** (Ellis 1983): how do we find the homogeneous model that best fits the inhomogeneous universe?
- Some regions are over-dense, some are under-dense w.r.t. average density.
    - Expansion of some regions will be faster than average expansion
    - **Result: average expansion will accelerate**