Gravitation, Gauge Theory and Quantum Physics

Ivan Debono

Institute of Astronomy and Astrophysics

8 February 2006

The Standard Model

- All physical interactions described by 4 fundamental forces:

Force	Field Particle	Theory
Gravity	graviton	General Relativity
Electromagnetism	photon	Electroweak Theory
Weak nuclear force	W and Z boson	
Strong nuclear force	gluon	Quantum Chromodynamics

Force	Coupling constant
Gravity	$\alpha_g = 10^{-39}$
Electromagnetism	$\alpha = 1/137$
Weak nuclear force	$\alpha_w = 10^{-6}$
Strong nuclear force	$\alpha_s = 1$

Typical laboratory energies ~ 1 eV

Planck energy ~ 10^{28} eV

Quantum gravity laboratory effects ~ 10^{-28} eV

General Relativity (Einstein, 1915)

- How do we include gravity within Special Relativity?

- Local Lorenz invariance
- General covariance
- Inertial motion is geodesic motion

- Equivalence Principle not compatible with Euclidean geometry of SR

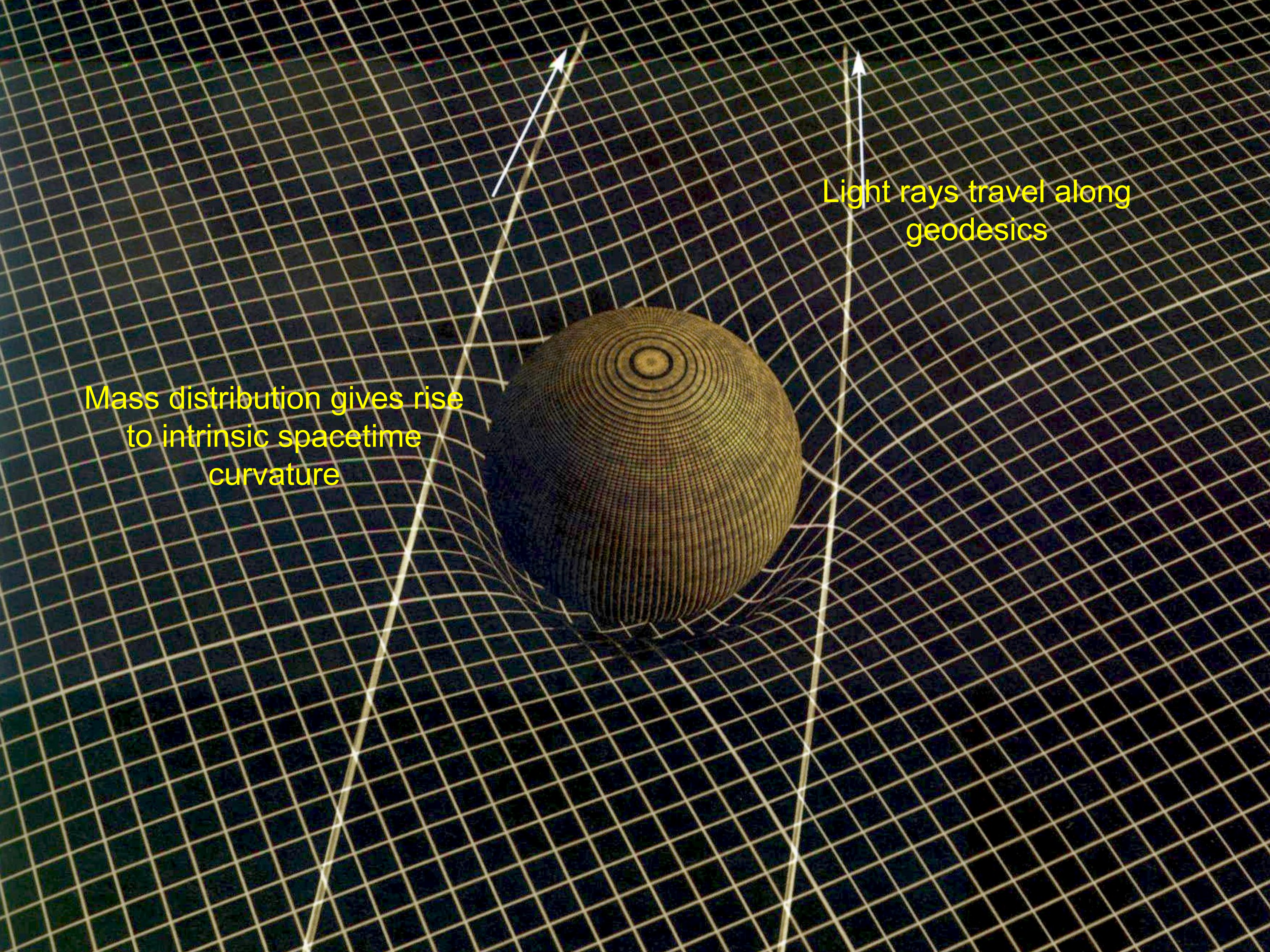
- Spacetime represented by 4-dimensional differentiable manifold (Riemannian)

- Manifold is equipped with Lorentz metric $g_{\mu\nu}$ giving spacetime distance between points

- Physical models consist of manifold, metric, and matter fields, which satisfy Einstein's Equation

$$G_{\mu\nu} = -\kappa T_{\mu\nu}$$

- Spacetime itself defines the field


Light rays travel along geodesics
Mass distribution gives rise to intrinsic spacetime curvature

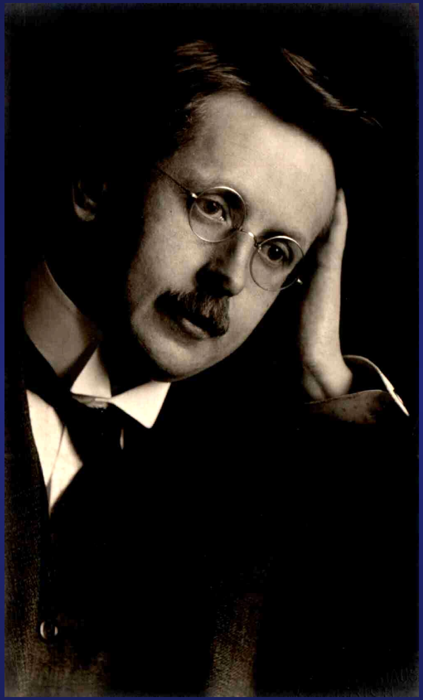
Can GR be quantized?

- Nonlinearity of Einstein’s field equations
  - Wave-particle duality requires linear vector space
- How do we quantize spacetime itself?
- How do we detect gravitons?

In Quantum theories, particle interactions are described by gauge theories.

The Gauge Principle

(Weyl, 1918; Yang & Mills, 1954)



- Length standards should be defined locally.
- Separate unit of length at each spacetime point
- Only relative lengths of any two vectors and angle between them is preserved under parallel transport

- Introduced gauge transformation

$$g_{\mu\nu}(x) \mapsto \lambda(x) g_{\mu\nu}(x)$$

- Attempted to unify gravity and EM

If a physical system is invariant with respect to some global (spacetime independent) group of continuous transformations G, then it remains invariant when G is considered locally (spacetime dependent) i.e. $G \longrightarrow G(x)$. Partial derivatives are replaced by covariant ones, which depend on some new vector field.

- These fields correspond to four known forces
- G is known as the gauge group.

- Yang & Mills:

Modern gauge theory

Electromagnetism as gauge theory

- Electromagnetic interactions arise from demanding invariance of quantum wave equation under local phase changes

- Perform phase transformation $\psi \rightarrow \psi' = \psi e^{i\alpha}$

- We can modify Schrödinger equation to make it invariant under local phase changes

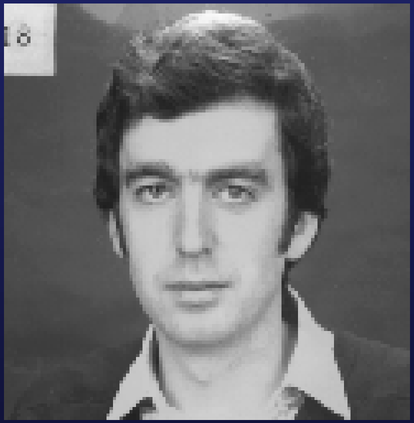
- Changes in a particle's phase result in changes to its momentum

- There must be a force which performs these changes

- That force is electromagnetism

Gauge Theory Gravity (GTG)
(Lasenby, Doran, Gull, 1990s)

- Motivated by attempt to find Theory of Gravity which follows gauge principle
- Aim: To model gravitational interactions in terms of gauge fields



Spacetime Algebra in GTG

- Utilise set of 4 orthonormal basis vectors satisfying $\gamma_\mu \cdot \gamma_\nu = \eta_{\mu\nu}$
- Set of 4 independent basis vectors for spacetime, giving 16-dimensional spacetime algebra
- Each point in vector space represented by vector $x = x^\mu \gamma_\mu$
- Background spacetime is global, as in other gauge theories
- Background spacetime is flat

Gauge Principles for Gravitation

- All physical fields have generic form $A(x) = B(x)$

A and B are spacetime fields
x is a spacetime algebra position vector

1. **Position Gauge Invariance**

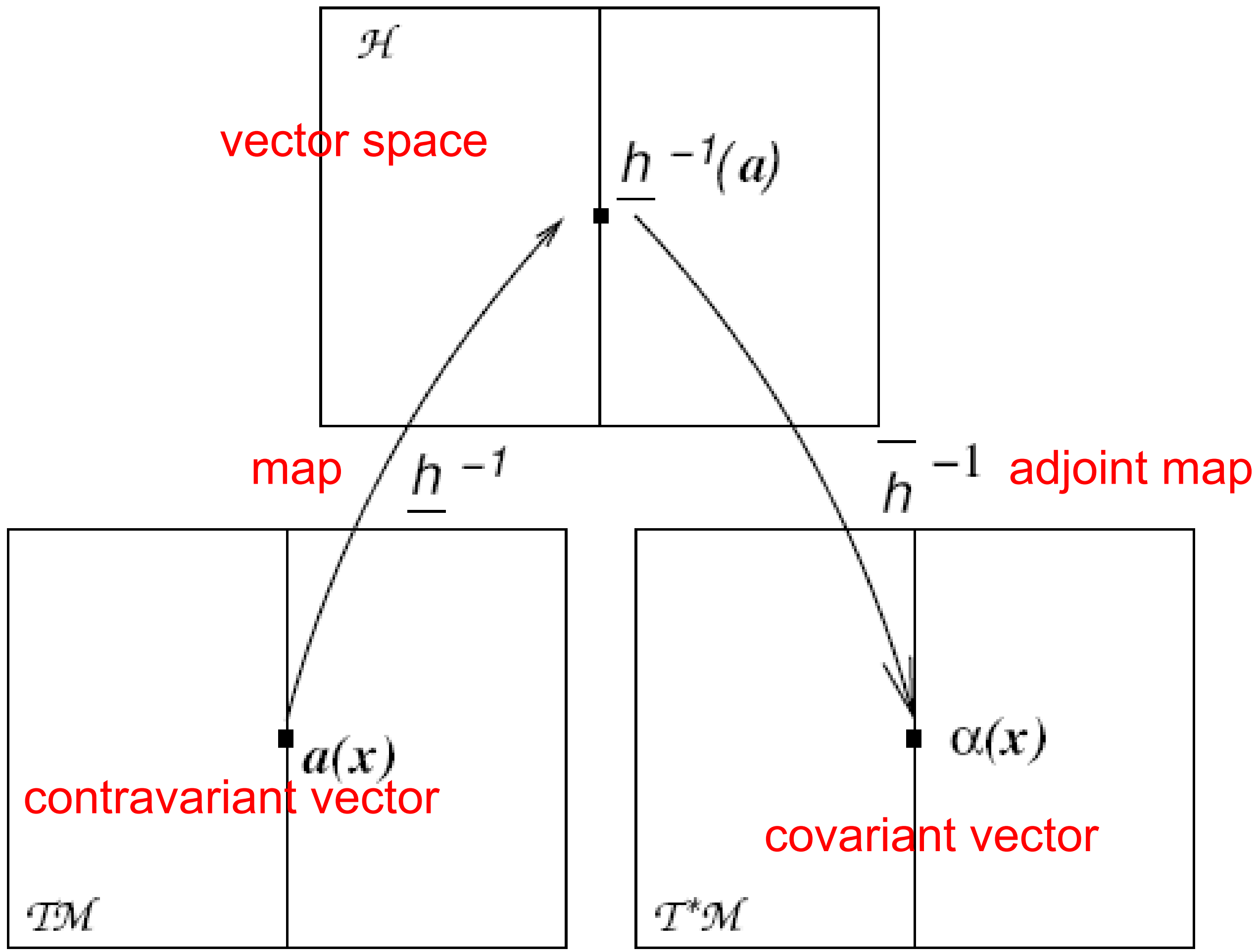
- Statement is independent of where we place the fields in spacetime algebra, i.e..
  If $x' = f(x)$ then $A'(x) = B'(x)$ has same physical content.

2. **Rotation Gauge Invariance**

- Intrinsic content of relation $A(x) = B(x)$ at a given x_o must be unchanged if we rotate A and B by same amount, i.e..
  $A'(x) = RA(x)$ and $B'(x) = RB(x)$ does not alter physical consequences.

Gravitation in GTG

- We need a field which guarantees gauge invariance
- Introduce new tensor field $\underline{h}^{-1}$ and its adjoint $\overline{h}^{-1}$
- Maps $\underline{h}^{-1}$ change the "metric" of spacetime
- Space $\mathcal{H}$ formed by maps plays role of "curved spacetime"
- "Gravitational force" is expressed via this field acting on flat spacetime
- Gauge group: set of local proper Lorentz transformations $SL(2,\mathbb{C})$



Metric in GTG : $g(e_\mu, e_\nu) = \underline{h}^{-1}(e_\mu) \cdot \underline{h}^{-1}(e_\nu) = g_\mu \cdot g_\nu.$

Field Equations

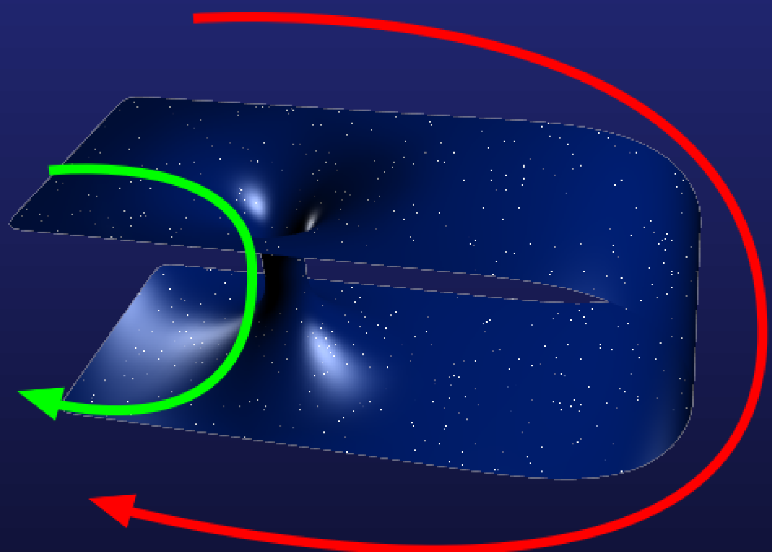
	GTG	GR
Ricci tensor:	$R_\mu = g^\nu \cdot R_{\nu\mu}$	$R_{\mu\nu} = g_\mu \cdot R_\nu = R_{\mu\lambda\nu}{}^\lambda$
Einstein tensor:	$R_\mu - \frac{1}{2} R g_\mu = -\kappa T_\mu$	$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu}$
Einstein Equation:	$G_\mu = -\kappa T_\mu$	$G_{\mu\nu} = -\kappa T_{\mu\nu}$

GR and GTG compared

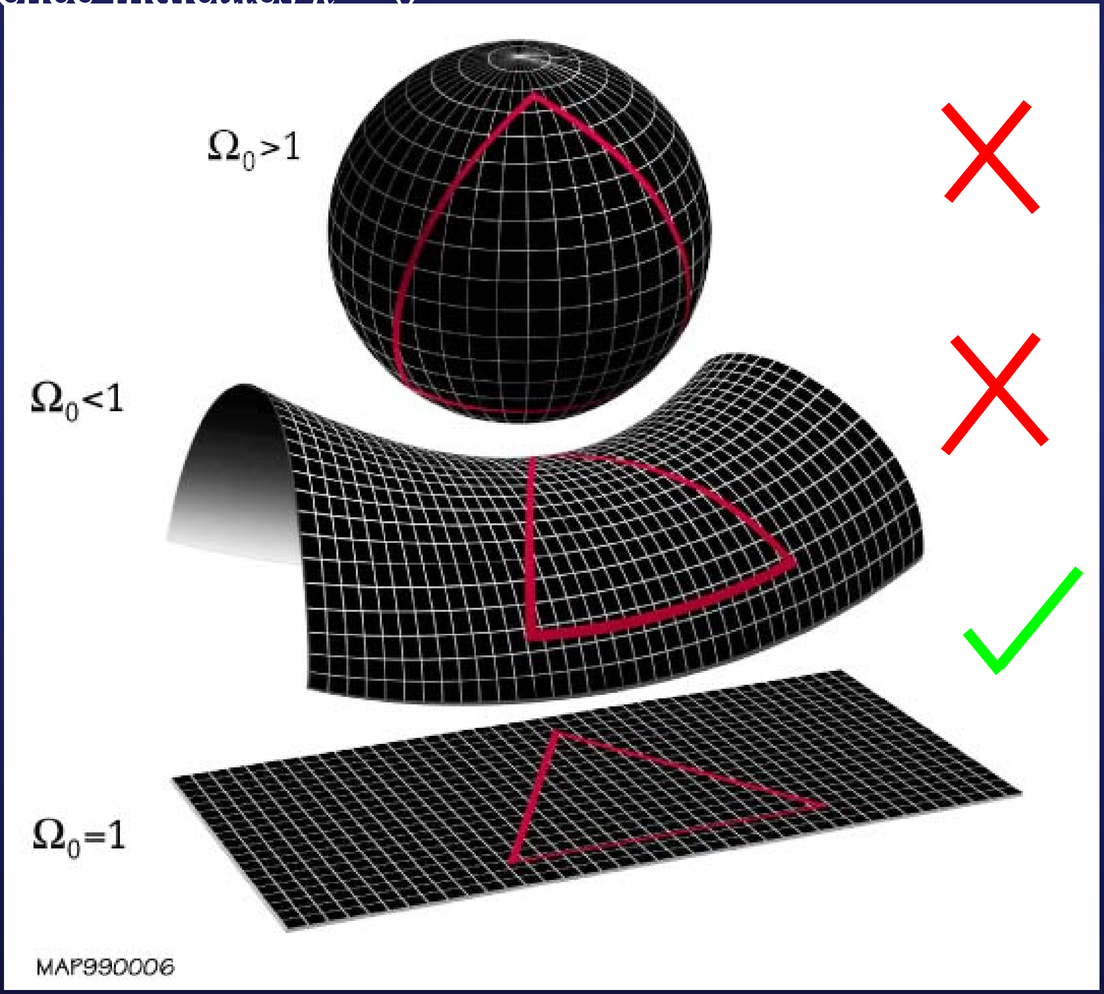
- GTG is locally equivalent to GR if spin effects are ignored
- Spin density is usually irrelevant on gravitational scales
- Theories agree in the Newtonian limit and Classical Tests of GR (Light deviation, perihelion shift, time delay, gravitational redshift)
- Theories shown to agree for most important physical models: static & rotating spherically symmetric, cylindrically symmetric matter distributions

Astrophysical and cosmological differences

- $\bar{h}^{-1}$ must be globally defined
- No time-symmetric black holes
- GTG does not allow "non-physical" solutions of Einstein's Equations (e.g. wormholes)
- No empirical evidence for such solutions

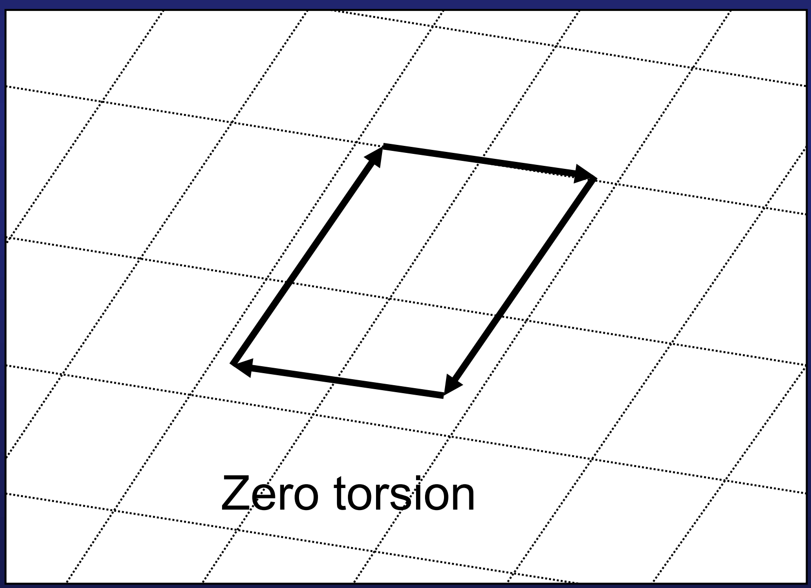


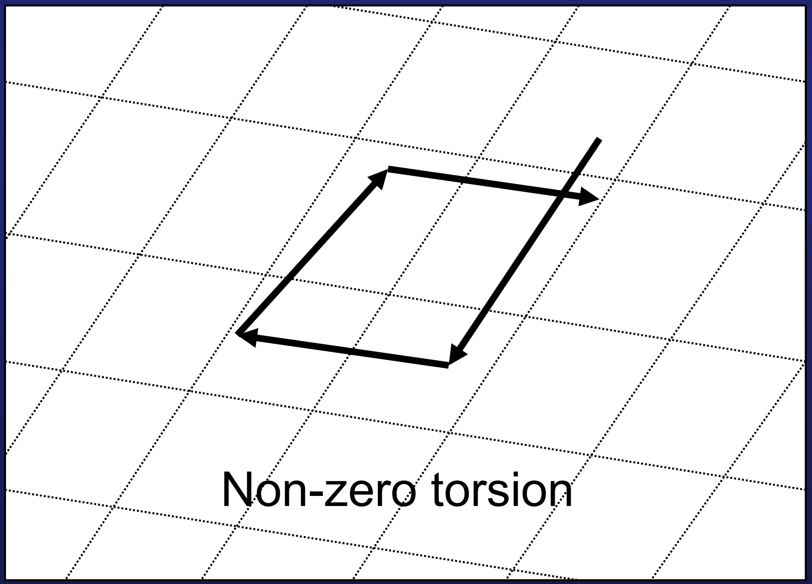
- Difficulties with $k \neq 0$ cosmological models because of global nature of curvature
- Current evidence indicates $k = 0$



GTG and spin

- GR (Riemannian geometry): Energy-momentum tensor $T_{\mu\nu}$ is symmetric
- But spin and orbital angular momentum cause tensor to be non-symmetric
- GR cannot properly model spin, and must be modified
- Affine torsion





- Spin contribution can be included in GTG since there is no *a priori* requirement for the energy-momentum tensor to be symmetric

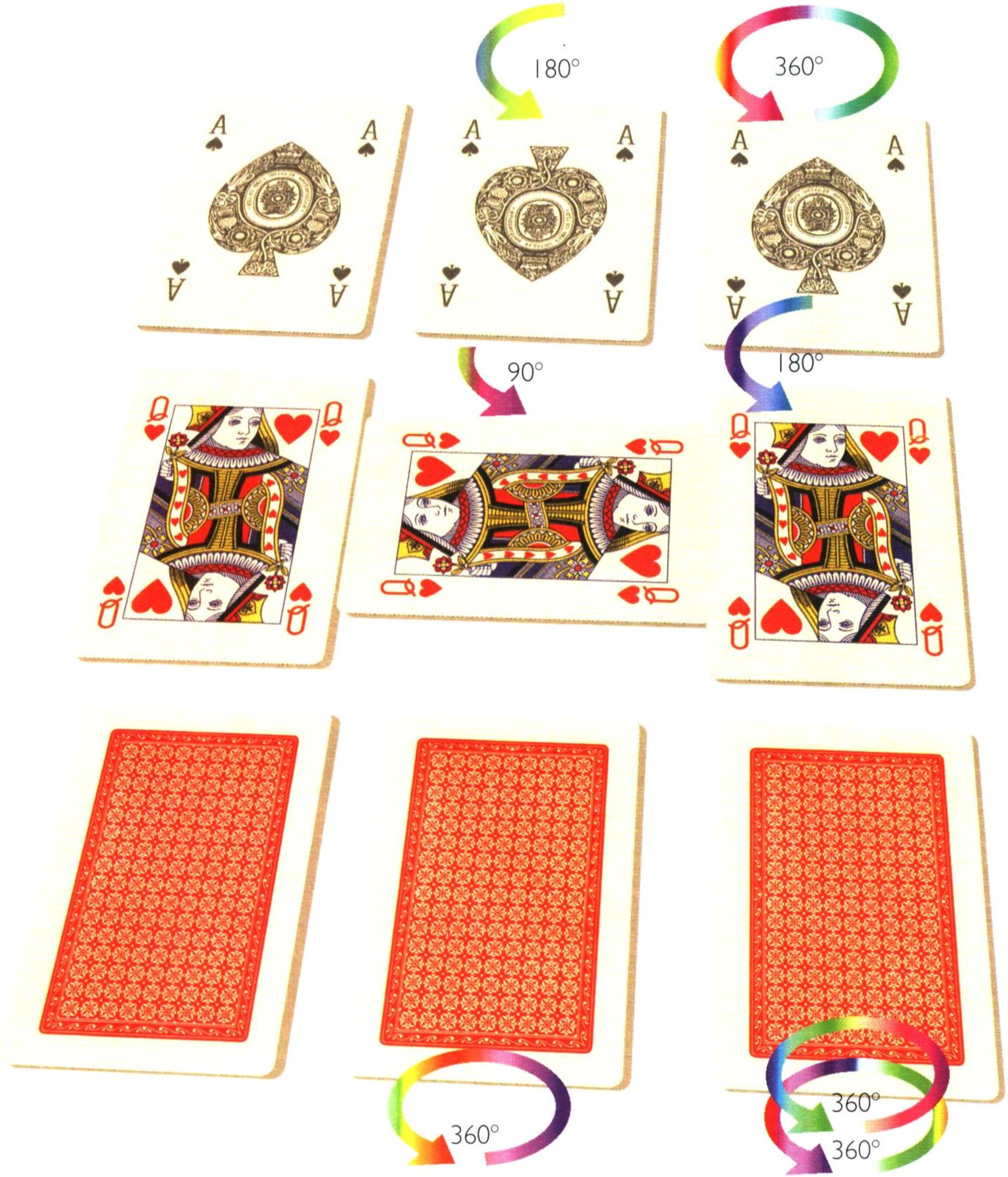
Spin

Macroscopic level:

- Fluid of rotating compact objects with strong gravitational field
  - Black holes
  - Neutron stars

Particle level:

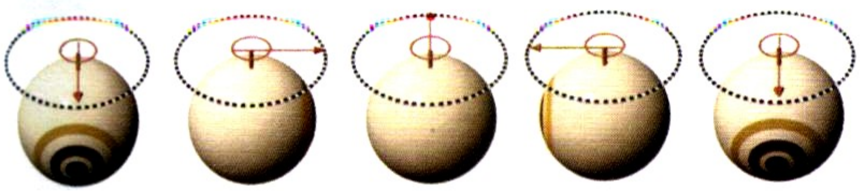
- Quantum spin
  - Particles in gravitational field



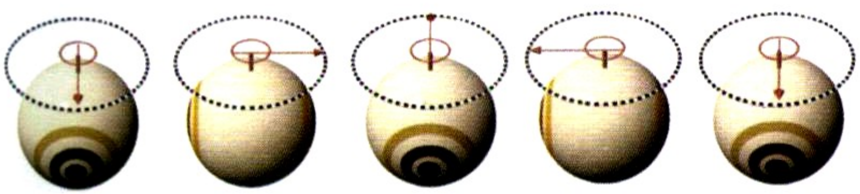
Particle with spin 1

Particle with spin 2

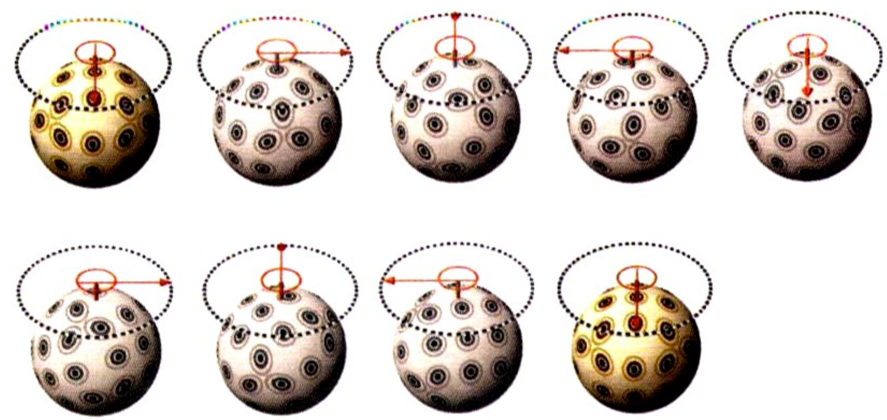
Particle with spin 1/2



Particle with spin 1



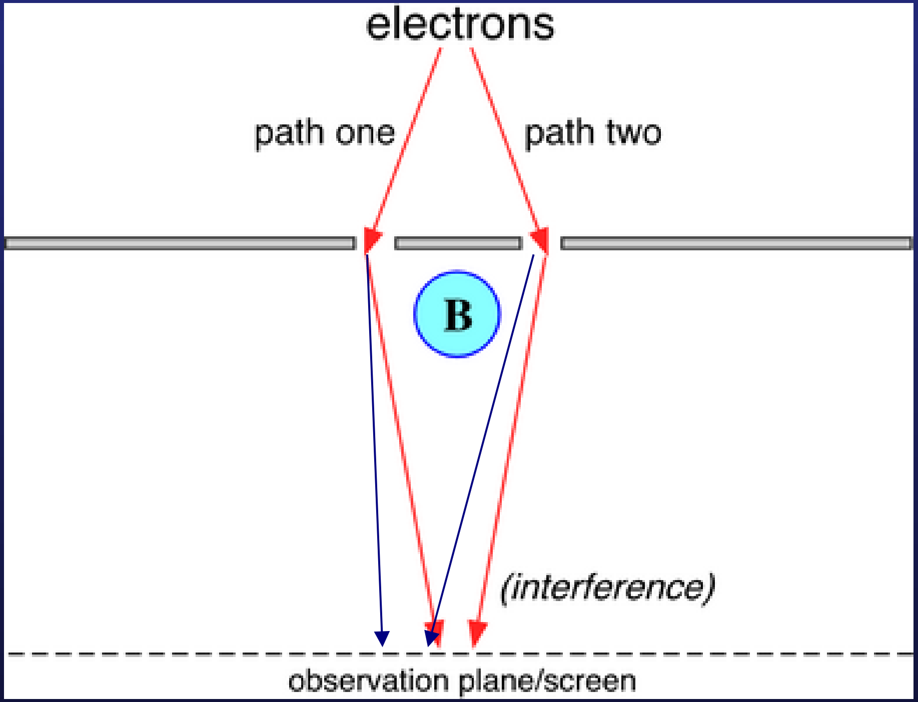
Particle with spin 2

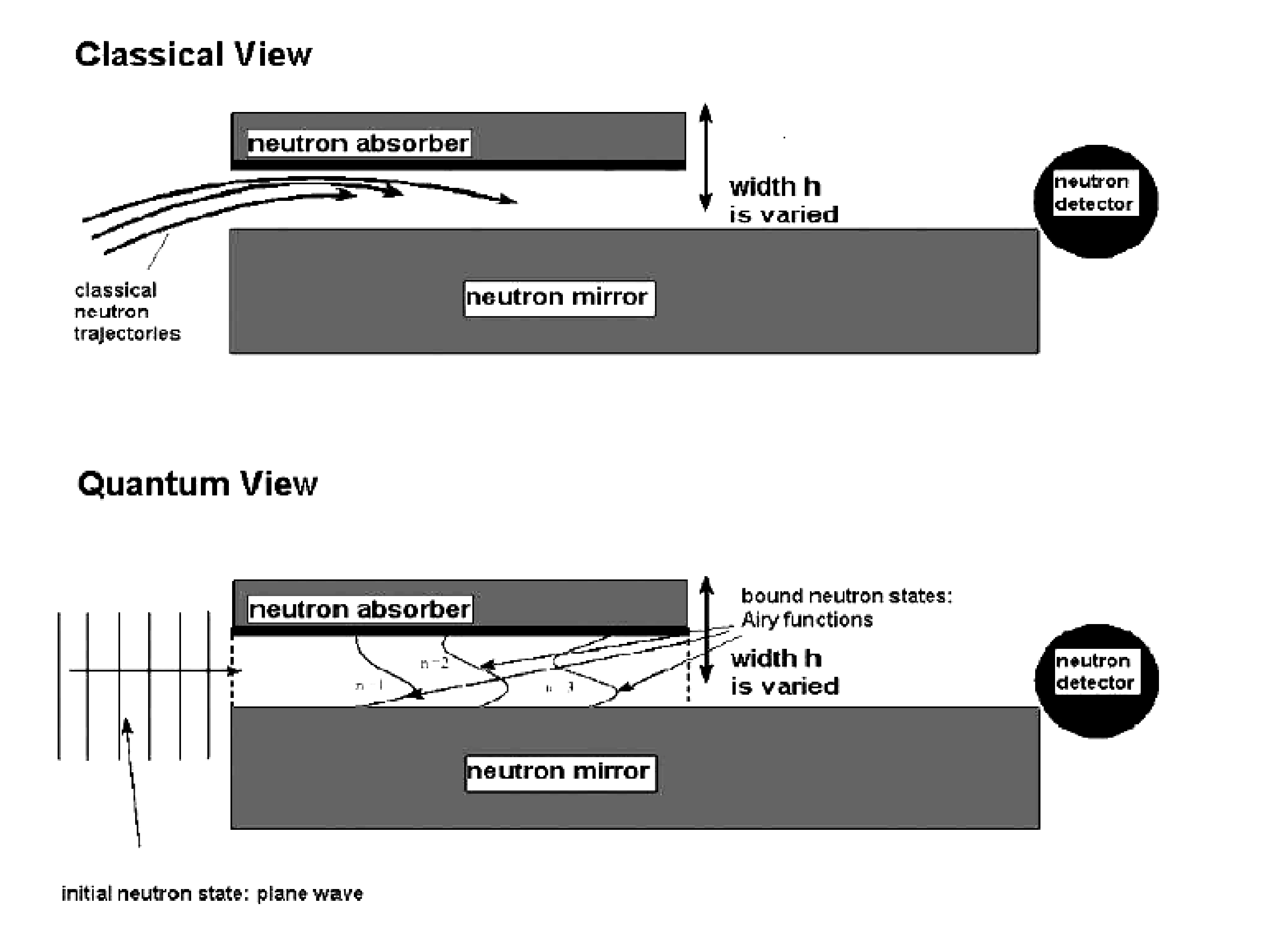


Particle with spin 1/2

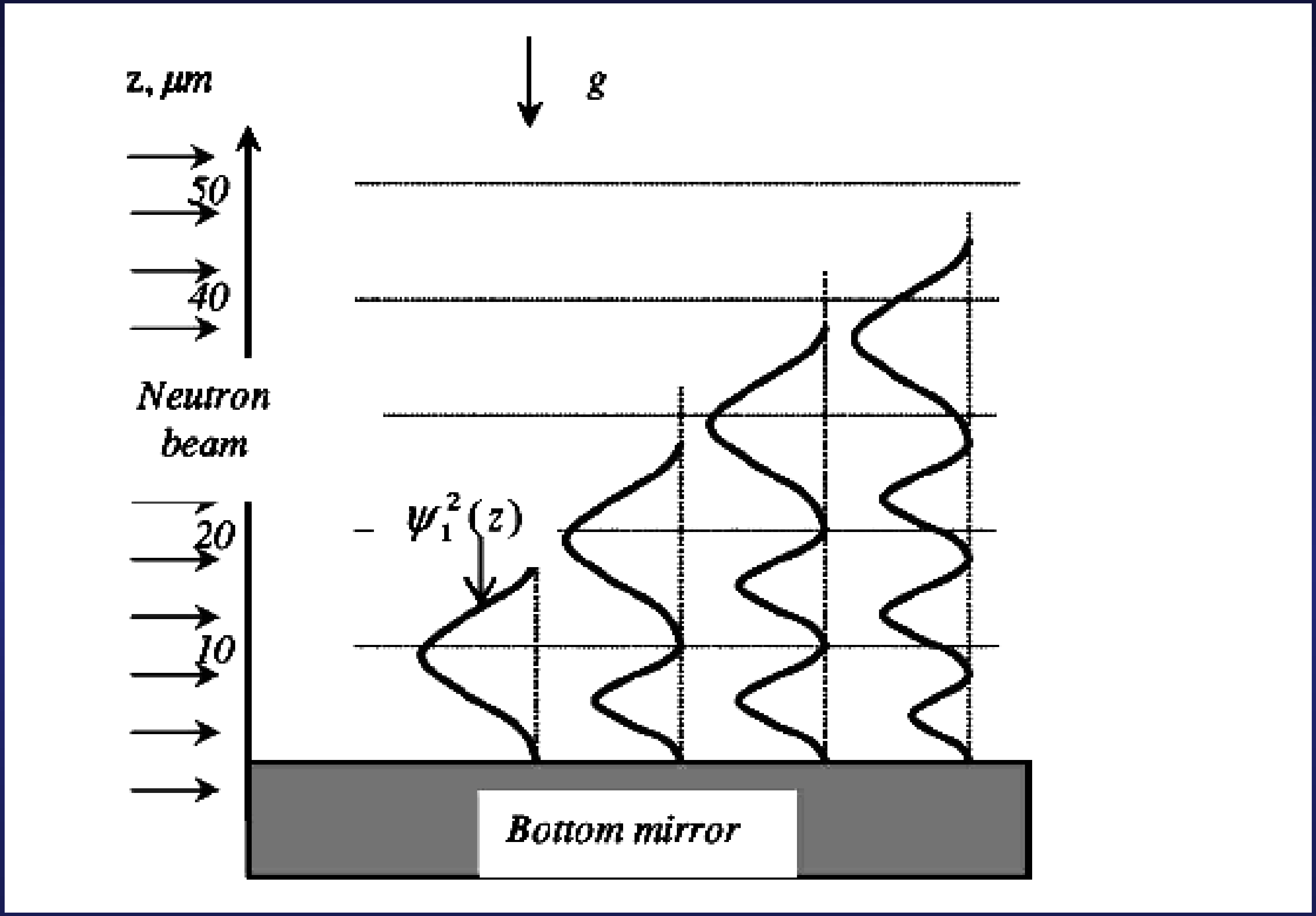
Quantum effects in gravitation

- Topology of gravitational gauge group (simply-connected) means that some quantum effects are not present
- e.g. Aharonov-Bohm effect not possible with gravitational field

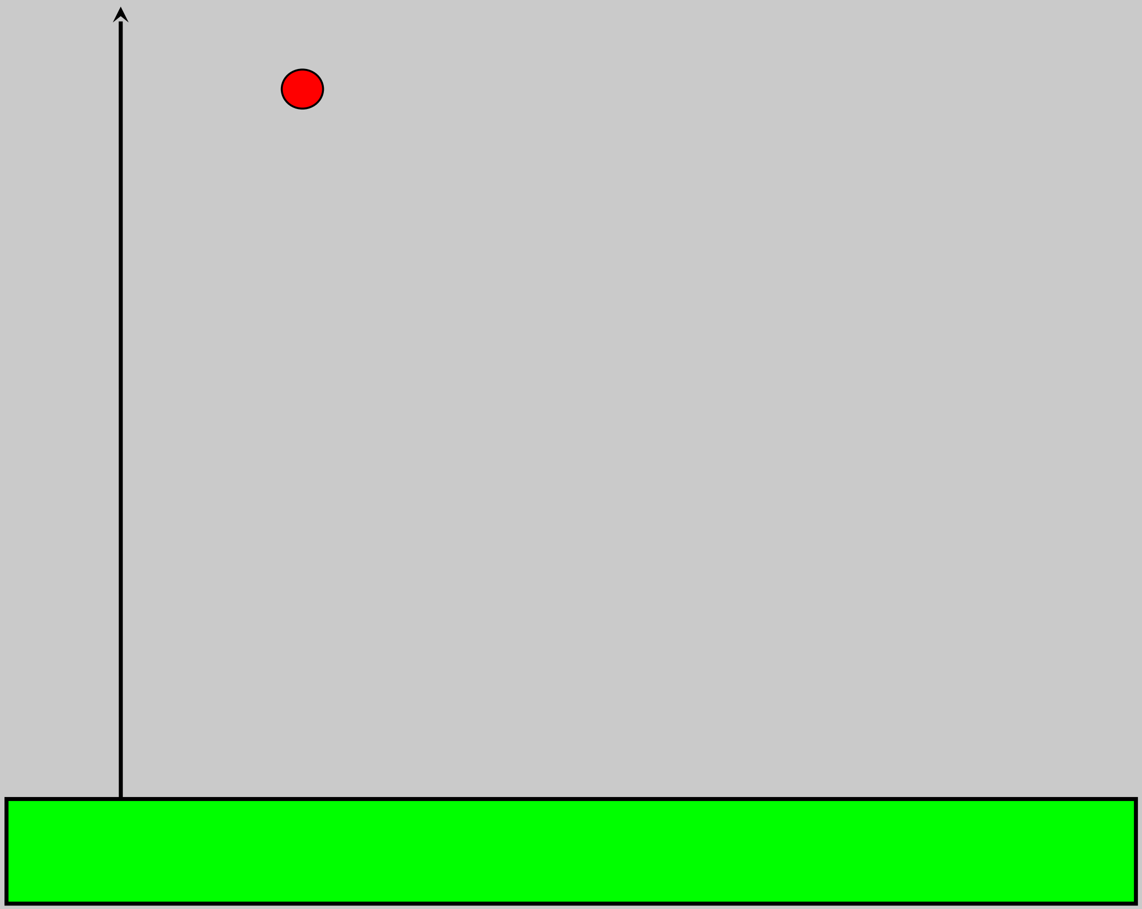


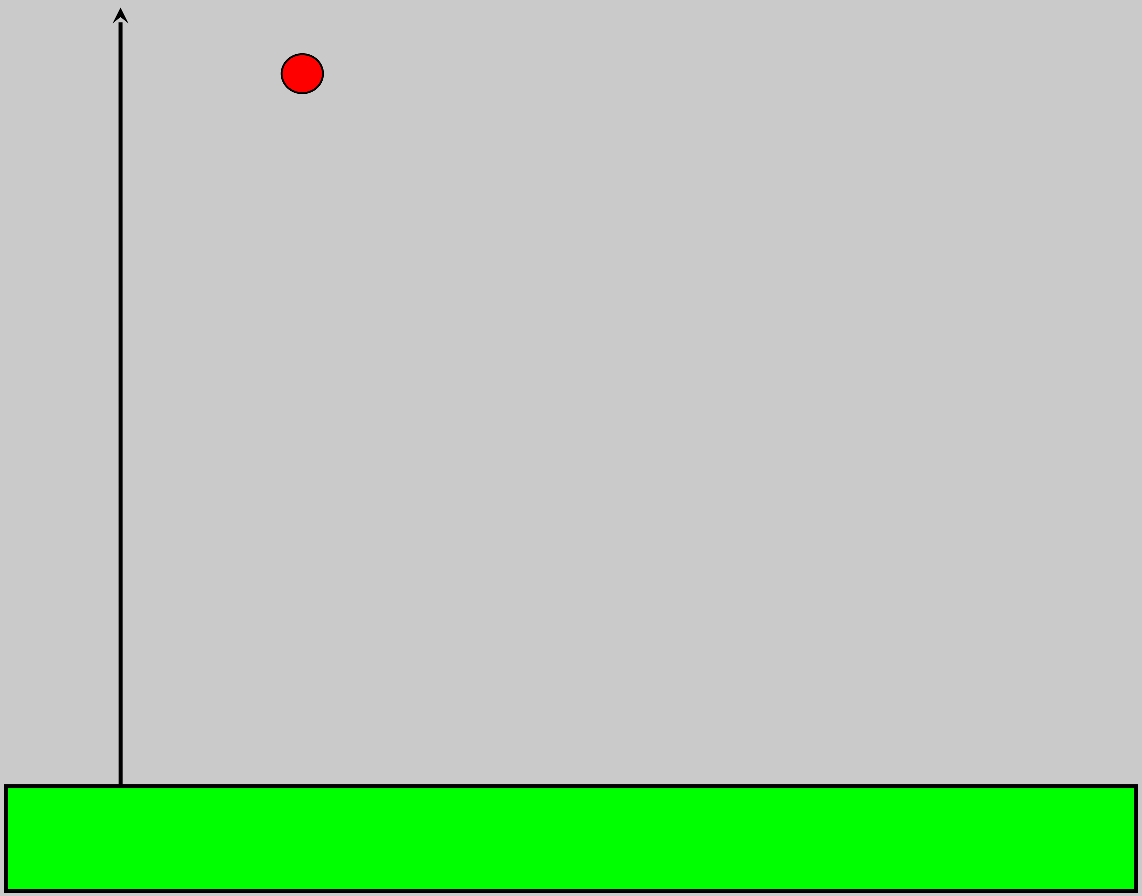


Abele, H., *et al.* (2003). Quantum states of neutrons in the gravitational field and limits for non-Newtonian interactions in the range between 1 µm and 10 µm, *Lect.Notes Phys.* 631, 355-366



Nesvizhevsky, V. V., *et al.* (2006). Measurement of quantum quantum states of neutrons in the Earth's gravitational field, *Phys. Rev.* D67, 102002.





Current Work

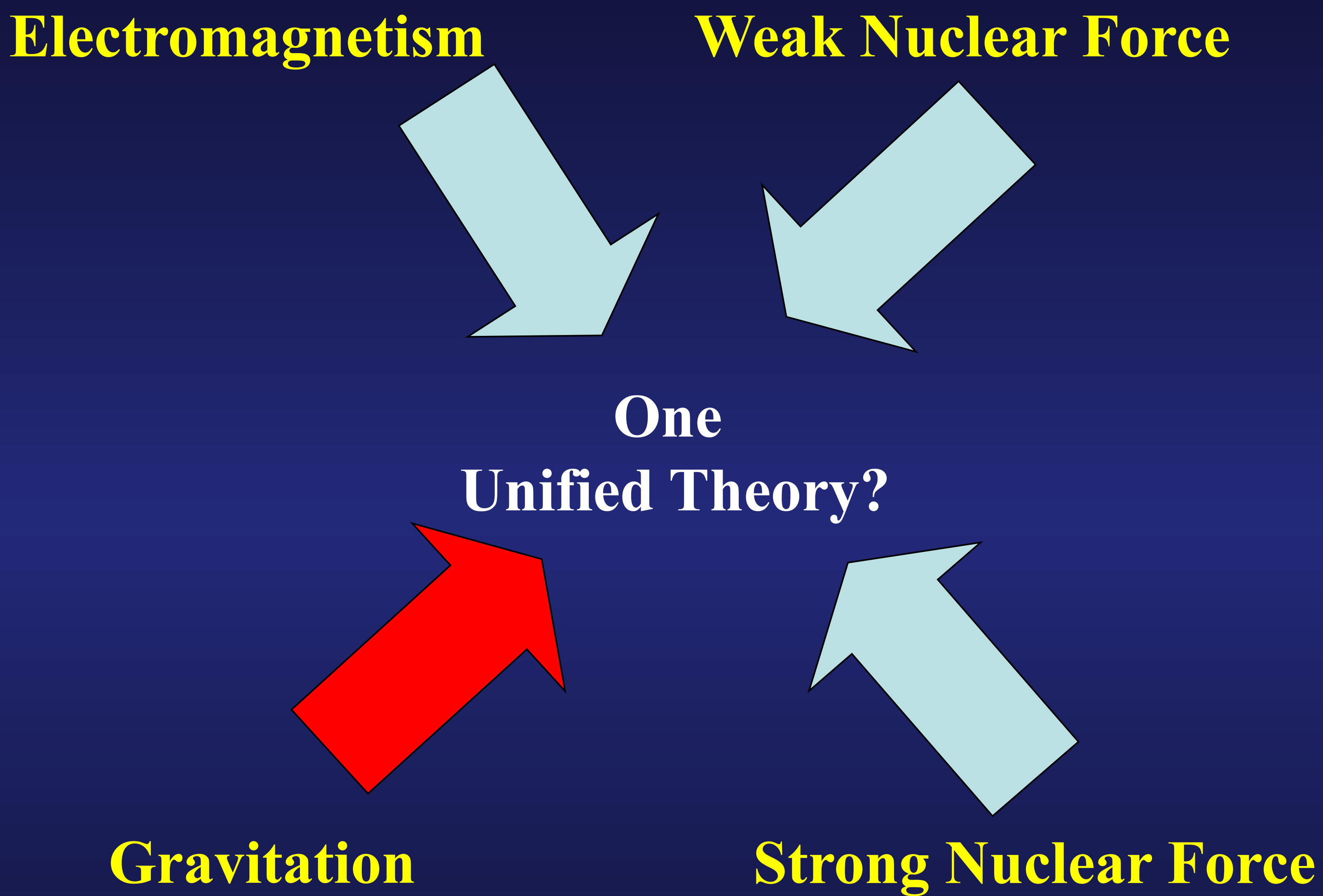
- **Finding an experiment which will differentiate between GR and GTG**

- Time asymmetric processes
- Black hole thermodynamics (Hawking radiation)

- Quantum spin effects
- Particles in gravitational background
  - Electron energy levels

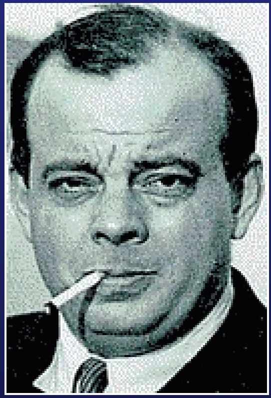
- Use of linearized theory in GTG
- Linearized theory of gravitational waves

- Exact solutions for gravitational waves
- Gravitational waves in “curved background spacetime”
- Solutions in strong-field region
- Comparison with GR results


Electromagnetism
Weak Nuclear Force
One
Unified Theory?
Gravitation
Strong Nuclear Force

Mon point de vue simplifie-t-il ou non l'univers? Et si j'invente une géométrie que rien d'objectif ne m'impose, ni me démontre, pure création de mon imagination, je la prends pour explication si elle me simplifie le monde.

Antoine de Saint-Exupéry



Does my point of view simplify the universe? And if I invent a geometry which is not objectively motivated, or demonstrated, created only by my imagination, I will use it as an explanation if it simplifies the world.